Simulation of Coupled Hydraulic Fracturing Propagation and Gas Well Performance in Shale Gas Reservoirs

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Abstract

Hydraulic fracturing combined with horizontal drilling has been the technology that makes it possible to economically produce natural gas from unconventional shale gas reservoirs. For best performance of fracturing stimulation, hydraulic fracturing designing parameters, such as different proppants, fracturing liquids, and injection rate and pressure, should be evaluated before the operation in a particular reservoir system. Traditional evaluation and optimization approaches are usually based on the simulated fracture properties along, such as fracture areas. In our opinion, the better methodology should be also related to production data from the stimulated wells afterwards, because the enhanced production is the ultimate goal.

In this paper, we present a general fully-coupled numerical framework to simulate the hydraulic induced fracture propagation and post-fracture gas well performance. This three-dimensional, multi-phase simulator focuses on: (1) fracture width increase and fracture propagation that occurs as slurry is injected into the fracture, (2) erosion caused by fracture fluid and leakoff, (3) proppant subsidence and flowback, and (4) multi-phase fluid flow through various-scaled anisotropic natural and man-made fractures. Mathematical and numerical details on how to fully couple the fracture propagation part and the fluid flow part are discussed. Fracturing and production operation parameters, properties of the formation, and reservoir, fluid, and proppant properties are all taken into account in this model. The well may be horizontal, vertical, or deviated, as well as open-hole or cemented. This simulator is verified based on benchmarks the literature. We show its application by simulating the fracture network (hydraulic and natural fractures) propagation and production data history matching of field production data in China. We conduct a series of real-data modeling studies with different combinations of fracturing parameters and present the methodology to design fracturing operations with feedback of simulated production data. This unified model aids in the optimization of hydraulic fracturing design, operations, and production.

Introduction

A fully coupled numerical framework to simulate the hydraulically induced fracture propagation and post-fracture gas well performance is important for the optimization of hydraulic fracturing design and oil/gas production. Generally, several interconnected physical processes need to be considered: (1) fracture width increase and fracture propagation occurring as slurry is injected into the fracture, (2) erosion caused by fracturing fluid and leakoff, (3) proppant subsidence and flowback, and (4) multi-phase fluid flow through various-scaled anisotropic natural and man-made fractures. In this paper, we introduce the theory and methodology for simulating fracture propagation and multi-phase flow separately, and afterwards a numerical framework to couple them.

The primary objective of the multi-phase flow portion is to develop an efficient reservoir simulator for modeling multidimensional gas and water flow and production from unconventional tight sandstone or shale gas reservoirs. Specifically, it includes: (1) developing conceptual models and improved modeling approaches for integrating hydrological, mechanical, and physical processes in hydraulically fractured tight sand and shale gas reservoirs; and (2) incorporating conceptual models into a numerical modeling tool for assessment and management of gas production from tight sand gas reservoirs using hydraulically fractured wells. The latter part of the developed simulator is able to quantify model physical processes and predict long-term performance of tight gas reservoirs for optimum gas production from these unconventional resources.
The goal of the fracture propagation portion is to develop a three-dimensional, single-well simulation model for use in design and evaluation of hydraulic fracturing processes in tight gas and shale gas reservoirs. The single well may be horizontal, vertical, or deviated, and may also be open-hole or cemented. The reservoir portion of the model would be the standard reservoir flow formulation based on Darcy’s law and conservation of mass. Such a model is characterized by rock porosity and permeability and fluid pressure and saturations. These rock properties will be spatially variable and fluid phases will be aqueous and gas.

Theory and Methodology

1) Fracture Mass and Flow Equations

We consider a fracture to be a thin, planar slit. The fracture width is in the y-direction and the fracture plane in the other two dimensions. Slurry, consisting of fluid and proppant components, flows through the fracture and the slurry components are conserved. The proppant conservation equation is:

\[
\frac{\partial \rho_p \tilde{v}_p c_p w}{\partial x} + \frac{\partial \rho_p \tilde{v}_p c_p w}{\partial z} + \frac{\partial \left[ \rho_p c_p w \right]}{\partial t} = 0
\] (1)

and the fluid component conservation equation is:

\[
\frac{\partial \rho_f \tilde{v}_f (1 - \Sigma c_p) w}{\partial x} + \frac{\partial \rho_f \tilde{v}_f (1 - \Sigma c_p) w}{\partial z} + \frac{\partial \left[ \rho_f x_f (1 - \Sigma c_p) w \right]}{\partial t} + \dot{q}_{\text{leak}} \rho_f x_f = 0
\] (2)

where \( c_p \) is proppant volume fraction, \( x_f \) is fluid volume fraction relative to total fluid, \( \rho \) is density, \( v \) is velocity, \( w \) is fracture width, and \( \dot{q}_{\text{leak}} \) is the fluid leakoff rate per unit area.

We assume fluid and proppant to be incompressible in the fracture. Then, densities in Equations 1 and 2 cancel. We sum those equations over proppant and fluid components, respectively, and add the two to obtain an overall slurry balance:

\[
\frac{\partial \left[ \Sigma p c_p \tilde{v}_p + (1 - \Sigma p c_p) \tilde{v}_f \right] w}{\partial x} + \frac{\partial \left[ \Sigma p c_p \tilde{v}_p + (1 - \Sigma p c_p) \tilde{v}_f \right] w}{\partial z} + \frac{\partial \left[ \Sigma p c_p \tilde{v}_p + (1 - \Sigma p c_p) \tilde{v}_f \right] \Sigma c_p w}{\partial t} + \dot{q}_{\text{leak}} = 0
\] (3)

We equate the coefficient of \( w \) in the derivative terms to the slurry velocity:

\[
\tilde{v}_{sl} = \Sigma p c_p \tilde{v}_p + (1 - \Sigma p c_p) \tilde{v}_f
\] (4)

and assume slurry velocity is that for laminar flow through a slit of width \( w \):

\[
\tilde{v}_{sl} = -\frac{w^2}{12 \mu_{st}} \nabla (P + \gamma z)
\] (5)

where \( P \) is fracture pressure, \( \mu_{st} \) is slurry viscosity, and \( \gamma \) is slurry gradient.

Slurry viscosity depends on proppant volume fraction as well as fluid component rheology. A number of expressions have appeared in the literature for the viscosity of suspensions containing solid particles (for example, Nicodemo et al., 1974) and we use the common exponential relation of the form:

\[
\mu_{st} = \mu_{fl} \left(1 - \frac{c_p}{c_{\text{max}}} \right)^{-n}
\] (6)

where \( \mu_{fl} \) is the fluid viscosity, \( c_{\text{max}} \) is the maximum proppant volume fraction at which slurry is essentially a solid porous medium, and \( n \) is an exponent typically between 1.0 and 2.5.

For z-direction flow, proppant particles settle because proppant is more dense than fluid; for x-direction flow, proppant velocity differs from slurry velocity due to inertial effects and the effect of slurry particles colliding with each other and the fracture wall. Friehauf (2009) presented expressions for these effects. The z-direction proppant, settling velocity has the form:

\[
v_{stlz} = v_{\text{stokes}} f(N_{re}) g(c_p) h(w)
\] (7)
where \( v_{stokes} \) is the Stoke’s settling velocity, \( f(N_{re}) \) captures inertial effects, \( g(c_p) \) captures the effect of interfering proppant particles, and \( h(w) \) captures the effect of the fracture wall on proppant velocity. The \( z \)-direction proppant velocity component is the sum of the slurry and settling velocity components:

\[
v_{p,z} = v_{sl,z} + v_{stlz} \tag{8}
\]

For \( x \)-direction flow, the fracture wall and other proppant particles both retard the proppant velocity:

\[
v_{p,x} = k(c_p, w) v_{slx} \tag{9}
\]

where \( k(c_p, w) \) is a function quantifying these effects.

The pressure difference between the fracture and the permeable reservoir causes fluid to leak off into the formation. Fluid leakoff is accompanied by the buildup of a filter cake at the fracture wall, a growing invaded zone of fracture fluid in the formation starting at the fracture wall, and the compression of the formation fluid from this growing invaded zone. Each of these leakoff mechanisms has been described by a leakoff coefficient (Schechter, 1992) and these leakoff coefficients are combined into an overall leakoff coefficient, \( C_{tot} \):

\[
C_{tot} = \frac{-\frac{1}{c_c} + \frac{1}{c_i} + \frac{1}{2} \left( \frac{1}{c_c^2} + \frac{1}{c_i^2} \right)}{\frac{1}{c_i^2} \cdot \frac{1}{c_c^2}} \tag{10}
\]

where \( C_p, C_v, \) and \( C_c \) are leakoff coefficients for the filter cake, invaded zone, and compressible formation, respectively, with fluid leakoff rate given by:

\[
\dot{q}_{\text{leak}} = \frac{2}{(\tau - t)} \tag{11}
\]

where \( \tau \) is the leakoff start time.

2) Fracture Mechanics Equations

Timoshenko and Goodier (1951) present a solution for the deflection caused by a load distributed over the boundary of a semi-infinite elastic medium. We calculate fracture width from their solution, with the fracture plane perpendicular to the minimum reservoir stress direction, the load given by the difference between the fracture pressure and the minimum reservoir stress, a factor of two added because there are two fracture faces, and rock properties in the integrand since they can vary over the fracture face:

\[
w(x, z) = 2 \int \frac{(1-v^2)}{E} (P - \sigma_{min}) r dr d\theta \tag{12}
\]

where \( E \) is Young’s modulus, \( v \) is Poisson’s ratio, \( P \) is fracture pressure, and \( \sigma_{min} \) is minimum reservoir stress. Barree and Conway (1994) have also used this approach to calculate fracture width.

Yew (1997) presents a fracture extension criterion. The stress intensity factor at the fracture tip is given by:

\[
K_I = \frac{E}{8(1-v^2)} \sqrt{\frac{2\pi}{r}} w(r) \tag{13}
\]

where \( r \) is the inward normal distance from the fracture front. The movement of the fracture front a distance \( d \) is given by

\[
d = \max \left\{ d_{max}, \frac{K_I - K_{IC}}{\sigma_{min} H_{frac}} \right\} \tag{14}
\]

where \( K_{IC} \) is the rock critical stress intensity factor, \( H_{frac} \) is the local fracture height, and \( z_{loc} \) is the relative depth of the fracture.
3) Wellbore Formulation

A wellbore is coupled to the fracture in order to model the transit of slurry from the surface to the fractures. It consists of a series of connected linear well segments, with each having a starting and ending measured depth and true vertical depth, and constant outer and inner radii. Slurry segments flow through the well in a piston-like manner and the fluid components of slurry are assumed to be slightly compressible. Figure 1 shows a schematic of a wellbore. There are two well segments, a vertical one and a horizontal one, and three slurry segments.

![Figure 1. Schematic of wellbore with two segments and three stages. Single arrow denotes injection and arrow triplet denotes fluid loss to the surroundings. MD denotes measured depth and Si denotes the slurry segment boundaries.](image)

A mass balance for fluid and proppant for injected slurry segment $i$ is:

$$\frac{\partial}{\partial t} \int_{S_{i+1}}^{S_i} \rho_x S_x dS + q_{loss} \rho_x S_x - q_{inj} \rho_x S_x = 0, \quad x = f, p$$

(15)

where segment $i$ is found from measured depth $S_{i+1}$ to $S_i$ along the wellbore, $A$ is the well segment cross sectional area, $q_{inj}$ is injection rate and the injection term is nonzero only for the slurry segment currently being injected, and $q_{loss}$ is rate of fluid loss to the fracture for completed well segments.

The pressure gradient along the wellbore has frictional and gravitational components:

$$\frac{\partial p}{\partial s} = \rho \mathbf{g} \cdot \mathbf{s} + \frac{\partial P_{fric}}{\partial s}$$

(16)

where $\mathbf{g}$ is the gravitational vector, $\mathbf{s}$ is the unit vector in the direction of the wellbore, and $P_{fric}$ is the frictional portion of the pressure gradient. This frictional portion is correlated with slurry Reynolds’s number.

4) Two-Phase Flow Model

The two phase flow model, consisting of gas and water (or liquid) in a porous or fractured unconventional reservoir, is similar to the two-phase black oil hydrocarbon model. For simplicity, the gas and water components are assumed to be present only in their associated phases and adsorbed gas is within the solid phase of rock. Each phase flows in response to pressure, gravitational, and capillary forces according to the multiphase extension of Darcy law or several extended non-Darcy flow laws, discussed below. In an isothermal system containing two mass components subject to multiphase flow and
adsorption, two mass-balance equations are needed to fully describe the system. For flow of phase $\beta$ ($g$ for gas and $w$ for water):

$$\frac{\partial}{\partial t}(\phi S_\beta \rho_\beta + m_\beta) = -\nabla \cdot (\rho_\beta \nu_\beta) + q_\beta$$ (17)

where $\phi$ is the effective porosity of porous or fractured media; $S_\beta$ is the saturation of fluid $\beta$; $\rho_\beta$ is the density of fluid $\beta$; $\nu_\beta$ is the volumetric velocity vector of fluid $\beta$, determined by Darcy’s law or non-Darcy flow models; $t$ is time; $m_\beta$ is the adsorption or desorption mass term for gas component per unit volume of rock formation; and $q_\beta$ is the sink/source mass term of phase (component) $\beta$ per unit volume of formation.

The Darcy velocity of phase $\beta$ is defined as:

$$\vec{V}_\beta = -\frac{k k_{\beta g}}{\mu_\beta} (\nabla P_\beta - \rho_\beta g \vec{D})$$ (18)

The governing Equations (17) of mass conservation for the two phases need to be supplemented with constitutive equations, which express all the parameters as functions of a set of primary thermodynamic variables of interest. The following relationships will be used to complete the statement of describing multiple phase flow of gas and liquid through porous and fractured media (Wang et al., 2013, Wu et al., 2013).

- **Saturation Constraint:**
  $$S_w + S_g = 1$$ (19)

- **Capillary Pressure Functions:**
  $$P_w = P_g - P_{cgw}(S_w)$$ (20)

  where $P_{cgw}$ is the gas-water capillary pressure in a two-phase system, which is assumed to be a function of water or gas saturation only.

- **Relative Permeability Functions:**
  $$k_{rw} = k_{rw}(S_w)$$ (21)
  $$k_{rg} = k_{rg}(S_g)$$ (22)

- **PVT Data:**
  $$\rho_w = \frac{(\rho_w)_{stc}}{B_w}$$ (23)
  $$\rho_g = \frac{Z_g M_g P}{RT}$$ (24)

  where $B_\beta$ is formation volume factor for phase $\beta$; $(\rho_\beta)_{stc}$ is density of phase $\beta$ at standard condition (or storage tank conditions); $M_g$ is average molecular weight; $Z_g$ is the $z$ factor to calibrate gas density from ideal gas to real gas; $R$ is universal gas constant.

- **Fluid Viscosities:**
  $$\mu_g = \mu_g(P_g)$$ (25)
• Formation Porosity:

$$\phi = \phi_0 \left(1 + C_T (P - P^0) - C_T (T - T^0)\right)$$  \hspace{1cm} (26)$$

where $\phi_0$ is the effective porosity of formation at reference pressure of $P_0$; and reference temperature, $T_0$; and $C_T$ is thermal expansion coefficient of formation rock.

5) Flowchart for the Coupled Fracture and Flow Model

![Flowchart of the Coupled Fracture and Flow Model](image-url)

Figure 2. Flowchart of the Coupled Fracture and Flow Model
Example Simulation and Discussion

In our fracturing simulation, there are two slurry fluid components called BASEFL and GEL. Both are Newtonian fluids with reference density 1020 kg/m³ and compressibility of $2 \times 10^{-10}$ Pa⁻¹. The viscosity of BASEFL is 0.001 Pa·s and the viscosity of GEL is 0.02 Pa·s. There is one proppant component with density 2800 kg/m³ and diameter 0.00045 m.

The well consists of two segments. The first one is a vertical segment from the surface to 3253 m measured depth (Figure 3). The second segment is a horizontal one 1713 m long to 4966 m measured depth. The outer diameter of the first segment is 9.5 in (0.24 m), the outer of the second is 6 in (0.15 m), and the inner diameter of both is 2.25 in (0.06 m). There are two open-hole completions, the first one from 4510 to 4530 m and the second from measured depth 4270 to 4290 m.

An injected stage contains 15 slurry segments. Information about these 15 segments in a stage is shown in Table 1. The fracture gridblocks are square with length 10 m. Reservoir compressibility is $10^{-9}$ Pa⁻¹ and reservoir viscosity is 0.0001 Pa·s. The maximum fracture front displacement parameter ($d_{max}$ in Equation 14) is 1 m. The system is isothermal, the wellbore initially contained BASEFL with no proppant, and the horizontal wellbore is parallel to the normal vector to the minimum stress plane. Consequently, each completed interval would have its own fracture. One slurry stage was injected into the first completed interval followed by another into the second completed interval with the first one.

Formation porosity is 0.772, permeability is $10^{-16}$ m², Young’s modulus is 10 GPa, Poisson’s ratio is 0.20, and tensile strength and toughness are both 5 MPa. Pressure is 10⁴ MPa at the surface and 31.9 MPa at the completion true vertical depth. Maximum stress is 79 MPa at 2253 m true vertical depth and 90.3 MPa at 3253 m true vertical depth. Minimum stress is 65 MPa at 2253 m true vertical depth and 87.3 MPa at 3253 m true vertical depth.

Figures 4 shows the fracture width profile and Figure 5 shows the proppant volume fraction profile at the end of the second injected stage for the second completed interval. The dot indicates the intersection of the horizontal well with the fracture. Injected proppant has been swept away from the vicinity of the well by fluid-only segment 15 in Table 1. Areas of 0.6 proppant fraction, the maximum proppant fraction, occur mostly at the fracture bottom due to proppant settling and fluid leakoff.
Table 1. Injected slurry composition for a stage.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Volume, m³</th>
<th>Injection Rate, m³/sec</th>
<th>Proppant Volume Fraction</th>
<th>Fluid Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>0.0417</td>
<td>0.0</td>
<td>BASEFL</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.06</td>
<td>0.0</td>
<td>GEL</td>
</tr>
<tr>
<td>3</td>
<td>8.2</td>
<td>0.06</td>
<td>0.0244</td>
<td>GEL</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>0.06</td>
<td>0.0</td>
<td>GEL</td>
</tr>
<tr>
<td>5</td>
<td>8.3</td>
<td>0.06</td>
<td>0.0361</td>
<td>GEL</td>
</tr>
<tr>
<td>6</td>
<td>43.0</td>
<td>0.06</td>
<td>0.0</td>
<td>GEL</td>
</tr>
<tr>
<td>7</td>
<td>15.6</td>
<td>0.06</td>
<td>0.0385</td>
<td>GEL</td>
</tr>
<tr>
<td>8</td>
<td>17.2</td>
<td>0.06</td>
<td>0.0698</td>
<td>GEL</td>
</tr>
<tr>
<td>9</td>
<td>21.1</td>
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<td>GEL</td>
</tr>
<tr>
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<td>0.06</td>
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<tr>
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</tr>
<tr>
<td>15</td>
<td>15.9</td>
<td>0.06</td>
<td>0.0</td>
<td>GEL</td>
</tr>
</tbody>
</table>

Figure 4. Fracture width profile at the end of the second stage for perforated interval 4270 to 4290 m.
For the two-phase flow model, wellbore pressure versus time is input, as shown in Figure 6, along with other data shown in Table 2 such as initial pressure, average porosity, gas saturation and so on. However, these data are not enough to specify the simulation and other parameters were estimated. Then, the calculated production rate is compared with the field data. They matched very well, indicating our simulation model can be applied to the analysis of field data, as shown in Figure 7.
### Table 2. Formation Evaluation Data

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
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<td>F</td>
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<tr>
<td>Effective permeability,</td>
<td>2.45</td>
<td>mD</td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>1.84E-2</td>
<td>cp</td>
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<tr>
<td>Porosity,</td>
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<td></td>
</tr>
<tr>
<td>Gas compressibility,</td>
<td>2.80E-4</td>
<td>1/psi</td>
</tr>
<tr>
<td>Rock compressibility,</td>
<td>3.45E-6</td>
<td>1/psi</td>
</tr>
</tbody>
</table>

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**Summary and Conclusions**

We developed a coupled hydraulic fracturing propagation and gas well performance model for shale gas reservoirs. Several important processes are considered in this model including: fracture width increase and fracture propagation that occurs as slurry is injected into the fracture; erosion caused by fracturing fluid and leakoff; proppant subsidence and flowback; and multi-phase fluid flow through various-scaled anisotropic natural and man-made fractures. We showed the mathematical and physical equations for the model and the methodology used to couple the two parts. We conducted an example study using field data. We show the fracture propagation and well performance, and the simulated results from this coupled model matched the production data very well.

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References


