Characterizing Hydraulic Fractures in Shale Gas Reservoirs Using Transient Pressure Tests
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Abstract
Hydraulic fracturing combined with horizontal drilling has been the technology that makes it possible to economically produce natural gas from unconventional shale gas reservoirs. Hydraulic fracturing operations, in particular, multistage fracturing treatments along horizontal wells in unconventional formations create complex fracture geometries or networks, which are difficult to characterize. The traditional analysis using a single vertical or horizontal fracture concept is no longer applicable. Knowledge of these created fracture properties, such as their spatial distribution, extension and fracture areas, is essential information to evaluate stimulation results. However, there are currently few effective approaches available for quantifying hydraulic fractures in unconventional reservoirs.

This work presents an unconventional gas reservoir simulator and its application to quantify hydraulic fractures in shale gas reservoirs using transient pressure data. The numerical model incorporates most known physical processes for gas production from unconventional reservoirs, including two-phase flow of liquid and gas, Klinkenberg effect, non-Darcy flow, and nonlinear adsorption. In addition, the model is able to handle various types and scales of fractures or heterogeneity using continuum, discrete or hybrid modeling approaches under different well production conditions of varying rate or pressure. Our modeling studies indicate that the most sensitive parameter of hydraulic fractures to early transient gas flow through extremely low permeability rock is actually the fracture-matrix contacting area, generated by fracturing stimulation. Based on this observation, it is possible to use transient pressure testing data to estimate the area of fractures generated from fracturing operations. We will conduct a series of modeling studies and present a methodology using typical transient pressure responses, simulated by the numerical model, to estimate fracture areas created or to quantify hydraulic fractures with traditional well testing technology. The type curves of pressure transients from this study can be used for quantify hydraulic fractures in field application.

Introduction
For the unconventional gas reservoirs, hydraulic fractures characterization is important in assuring that maximum efficiency is achieved for an environmental remediation system (Miskimins, 2009). Many papers have been published on behaviors of finite-conductivity or infinite-conductivity vertical fractures and their type-curves. Cinco-ley and Samaniego summarized that flow for a vertical fractured well could be divided into four periods: fracture linear flow, bilinear flow, formation linear flow and pseudo-radial flow (Cinco-ley and Samaniego, 1981; Baree et al. 2009; Apaydin et al. 2012). Some other work pointed out the dominant flow regime observed in most fractured tight/shale gas wells is linear flow, which may continue for several years (Nobakht and Clarkson, 2012). Transient pressure analysis of this period could provide lots useful information especially the total hydraulic fracture and matrix contact area. However, all these gas transient pressure analysis use the concept of pseudo pressure which is proposed for the conventional gas reservoir (Al-Hussainy and Ramey, 1965). For gas flow in unconventional reservoir, our work shows that gas-slippage effect and adsorption played important parts which cannot be neglected (Wu et al. 2013). To the authors’ knowledge, these two factors were not taken into consideration during the traditional pseudo pressure derivation. Therefore, a new pseudo pressure definition will be given and we will show how to estimate the total effective hydraulic fracture and matrix contact area.

This paper presents our continual effort in developing simulation models and tools for quantitative studies of unconventional
gas reservoirs (Wu and Fackahrenphol, 2011; Wu et al. 2012). Specifically, we explore the possibility of performing well testing analysis combining the numerical modeling and analytical approaches. The numerical model simulates realistic unconventional reservoir gas flow processes, including Klinkenberg effect, non-Darcy flow behavior, adsorption and geomechanics under single-phase and two phase flow condition. In addition, we also include wellbore storage and skin effect in gas production wells in the pressure transient analysis. We use the numerical model to generate type-curves for gas flow for well testing analysis in unconventional porous and fractured reservoirs with horizontal well and multistage hydraulic fractures. The type curves of pressure transients from this study can be used to quantify hydraulic fractures in field application.

**Derivation of New Pseudo Pressure**

In 1965, Al-Hussainy and Ramer derived the pseudo pressure which can be used successfully to analyze the flow of real gases.

\[
 m(P) = 2 \int_{P_0}^{P} \frac{P'}{\mu Z} dP'
\]

where \( P_0 \) is referene pressure; \( P \) is gas pressure; \( \mu \) is the gas viscosity and \( Z \) is gas pressure Z factor.

The concept of the real gas pseudo-pressure promises a considerable simplification and improvement in all phases of gas well analysis and gas reservoir calculations. These analysis and calculations worked very well for the conventional reservoirs but could have some trouble when used in the unconventional reservoirs analysis directly. This is because gas flow in ultra-low permeability unconventional reservoirs is subject to more nonlinear, coupled processes, including nonlinear adsorption/desorption, non-Darcy flow (at high flow rate and low flow rate), and strong rock-fluid interaction, and rock deformation within nanopores or micro-fractures, coexisting with complex flow geometry and multi-scaled heterogeneity.

Considering the Klinkenberg effects and gas adsorption, the principle of conversation of mass for isothermal gas flow through a porous media is expressed by the expression:

\[
 \nabla \cdot \left( \rho \frac{k(P)}{\mu(P)} \nabla P \right) = \frac{\partial}{\partial t} \left( \phi \rho + \frac{m_g(V)}{V} \right)
\]

The pressure-dependent permeability for gas was expressed by Klinkenberg as:

\[
 k_g = k_\infty \left( 1 + \frac{b}{P_g} \right)
\]

where \( k_\infty \) is constant, absolute gas-phase permeability under very large gas-phase pressure (where the Klinkenberg effect is minimized); and \( b \) is the Klinkenberg b-factor, accounting for gas-slippage effect.

The mass of adsorbed gas in formation volume, \( V \), is described (Leahy-Dios et al. 2011; Wu et al. 2012):

\[
 m_g(V) = \rho_k \rho_g f(P) V
\]

where \( m_g(V) \) is absorbed gas mass in a volume \( V \), \( \rho_k \) is rock bulk density; \( \rho_g \) is gas density at standard condition; \( f(P) \) is the adsorption isotherm function. If the adsorbed gas terms can be represented by the Langmuir isotherm (Langmuir, 1916), the dependency of adsorbed gas volume on pressure at constant temperature is given below,

\[
 f(P) = V_L \frac{P}{P + P_L}
\]

where \( V_L \) is the gas content or Langmuir’s volume in scf/ton (or standard volume adsorbed per unit rock mass); \( P \) is reservoir gas pressure; and \( P_L \) is Langmuir’s pressure, the pressure at which 50% of the gas is desorbed.

For real gas,
\[ P = \frac{M}{RT} \left[ \frac{P}{Z(P)} \right] \]  

(6)

Substitute Equations (3), (4), (5), (6) into Equation (2),

\[ \nabla \cdot \left[ \frac{P(1 + \frac{b}{P})}{\mu(P)Z(P)} \nabla P \right] = \frac{\phi}{k_e} \frac{\partial}{\partial t} \left[ \frac{P}{Z(P)} + \frac{RT\rho_k\rho_g V_L P}{\phi M(P + P_L)} \right] \]  

(7)

From the definition of the isothermal compressibility of gas:

\[ c_g(P) = \frac{Z(P)}{P} \frac{d}{dP} \left[ \frac{P}{Z(P)} \right] \]  

(8)

We also define the “compressibility” from the adsorption:

\[ c_a(P) = \frac{Z(P)}{P} \frac{d}{dP} \left[ \frac{RT\rho_k\rho_g V_L P}{\phi M(P + P_L)} \right] = \frac{Z(P)}{P} \frac{RT\rho_k\rho_g V_L P}{\phi M(P + P_L)^2} \]  

(9)

Let the total compressibility:

\[ c_t(P) = c_a(P) + c_g(P) \]  

(10)

Equation (7) will be:

\[ \nabla^2 P^2 - \frac{1}{dP^2} \left( \nabla P^2 \right)^2 = \frac{\phi \mu(P)c_t(P)}{k(1 + b/P)} \frac{\partial^2}{\partial t} \]  

(11)

Assume the viscosity and gas law deviation factors change slowly with pressure changes; the second part of Equation (11) becomes negligible. Equation (11) becomes:

\[ \nabla^2 P^2 = \frac{\phi \mu(P)c_t(P)}{k(1 + b/P)} \frac{\partial^2}{\partial t} \]  

(12)

We define the new pseudo-pressure \( m(P) \) as follows:

\[ m(P) = 2 \int_{P_0}^{P} \frac{P'(1 + b/P)}{\mu(P)Z(P)} dP' \]  

(13)

Equation (2) can be rewritten in terms of variable \( m(P) \) using the definition of \( c_t(P) \) given by Equations (8), (9) and (10) as

\[ \nabla^2 m(P) = \frac{\phi \mu(P)c_t(P)}{k(P)} \frac{\partial m(P)}{\partial t} \]  

(14)

Based on Equation (14), we could apply this form of the flow equation, quasi-linear flow equation, to the analysis of real gas flow behavior in unconventional reservoirs.
Linear Gas Flow
The primary flow regime observed in fractured tight/shale gas wells may be approximated to linear flow, which may continue for several years. Sometimes decline curve may show outer boundary effects but no pseudo-radial flow.

Wattenbarger et al. gived the “short-term” approximations for this linear flow with constant rate (Wattenbarger et al. 1998), respectively,

\[ m_{Di} = m_{Dwf} = qB \frac{\pi \mu}{\phi c_t \sqrt{kA} \sqrt{t}} \]  

(15)

In Equation (15), \( m_D \) is the pseudo pressure; \( q \) is the gas rate; \( B \) is the gas FVF; \( \phi \) is the formation porosity; \( c_t \) is the total compressibility, \( k \) is the formation permeability; \( A \) is the hydraulic fracture area and \( t \) is the time; subscript \( i \) refers to initial condition and subscript \( wf \) refers to the wellbore condition;

It showed that for the constant-flowing-rate boundary condition, linear flow appears as a straight line on the plot of normalized pressure vs. the square root of time. The slope of this square-root-of-time plot provides some useful information for estimate the hydraulic fracture area. The accuracy of this estimation is influenced by initial pressure, formation average permeability and total compressibility. (Nobakht and Clarkson, 2012)

The assumption of this model is one single hydraulic fracture with infinite conductivity neglecting the gas adsorption and wellbore storage effects. For the gas flow analysis in unconventional reservoirs, these assumption is generally unacceptable. Our work shows that adsorbed gas could contribute more than 30% to the total production in some shale fields (Wu et al. 2013). One way to include the adsorption into this model is adding “adsorption compressibility” to the total compressibility, as discussed above. For this model, considering gas adsorption will be more applicable and accurate. Otherwise, it will lead to an overestimation of the total hydraulic fracture area.

With our definition of the “adsorption compressibility” in Equation (9), an estimation of this adsorption compressibility is calculated shown below. Fig. 1 shows the gas Z factor value from 2000 psi to 5000 psi. Table 1 lists the data for our calculation.

**Table 1 Data used for the estimation of the “adsorption compressibility”**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas type</td>
<td>Methane</td>
<td></td>
</tr>
<tr>
<td>Porosity, ( \phi )</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Temperature, ( T )</td>
<td>122</td>
<td>°F</td>
</tr>
<tr>
<td>Rock density, ( \rho_K )</td>
<td>2.7</td>
<td>g/cc</td>
</tr>
<tr>
<td>Langmuir’s volume, ( V_L )</td>
<td>218.57</td>
<td>Scf/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure, ( P_L )</td>
<td>2285.7</td>
<td>psi</td>
</tr>
</tbody>
</table>

Fig. 2 is the calculated adsorption compressibility value and this value is in the same magnitude with the rock and fluids compressibility. From this figure we could see with the pressure increase from 2000 psi to 5000 psi, this compressibility drops from \( 3.16 \times 10^{-4} \) psi to \( 4.84 \times 10^{-5} \) psi. If we assume the rock and fluid compressibility value is constant as \( 2.5 \times 10^{-4} \) psi, the adsorption compressibility addition occupied from 19% to 126%. 
Apart from the influence of the “adsorption compressibility”, we will also analyze the influence of multi-stage hydraulic fractures and wellbore storage effect.

**Numerical Model**

The unconventional oil/gas reservoir simulator developed coupled multiphase fluid flow and mass diffusion with effect of rock deformation, Klinkenberg effect, non-Darcy flow and chemical reaction of adsorption and desorption processes in unconventional reservoirs. In numerical formulation, the integral finite difference method (Wu 1998; Pruess et al. 1983) is used for space discretization of multidimensional fluid and heat flow in porous and fractured reservoirs using an unstructured/structured grid. Time is discretized fully implicitly as a first-order backward finite difference. Time and space discretization of mass balance equations results in a set of coupled non-linear equations, solved fully implicitly using Newton iteration. (Wu et al. 2012 and 2013).

In our model, a hybrid-fracture modeling approach, defined as a combination of explicit-fracture (discrete fracture model), MINC (Multiple Interacting Continua) approach on the stimulated zones, and single-porosity modeling approaches on unstimulated areas (Fig. 2 and 3), is used for modeling a shale gas reservoir with both hydraulic fractures and natural fractures (Mayerhofer et al. 2010; Warpinski et al. 2008; Cipolla et al. 2010). This is because hydraulic fractures, which have to be dealt with for shale gas production, are better handled by the explicit fracture method. On the other hand, natural fractured reservoirs are better modeled by a dual-continuum approach, such as MINC for extremely low-permeability matrix in shale gas formations, which cannot be modeled by an explicit fracture model.
Fig. 3 shows our combination conceptual model for horizontal well, hydraulic fractures and stimulated reservoir volume (SRV). We assume the stimulated reservoir volume (SRV) is the area near the hydraulic fractures with natural fractures and we apply the MINC concept in this area. This SRV is shown in Fig. 3. A single-porosity model is applied in the region outside the SRV.

Refining the grids is also done for simulation of hydraulic fractures and their nearby regions, so that the true geometry of the fractures is modeled. In this method, fractures are represented by very fine gridblocks using actual fracture geometric data; properties of the fractures, such as permeability, are assigned to those fracture blocks.

Applications
In this section, we use the mathematical model for modeling transient pressure responses versus fracture areas during gas production from a shale gas reservoir. Here we present two gas flow problems to illustrate gas flow and transient pressure behavior in fractured wells. One is flow into a fractured horizontal well with multiple hydraulic fractures without SRV and the other with SRV.

\[
\frac{C_{fd}}{k} = \frac{k_f w_f}{k_m x_f} = 500
\]

Three different fracture models with the same total fracture-matrix contact area are built, as shown in Fig. 5, Fig. 6 and Fig. 7. Fig. 8 shows that transient pressure behaviors for these three cases in the earlytime almost remain the same and indicates that fracture number and fracture geometry are non-sensitive parameters of hydraulic fractures, as long as the total fracture area is.

Table 2. Data used for the case study and discussion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Reservoir length, $\Delta x$, ft</td>
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<tr>
<td>Reservoir width, $\Delta y$, ft</td>
<td>2000</td>
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<tr>
<td>Formation thickness, $\Delta z$, ft</td>
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<td>Production rate, $Q$, Mscf/day</td>
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<td>Hydraulic fracture width, $w_f$, ft</td>
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<td>Hydraulic fracture half-length, $X_f$, ft</td>
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</tr>
<tr>
<td>Matrix permeability, $k_m$, md</td>
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<tr>
<td>Matrix porosity, $\Phi_m$</td>
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<td></td>
</tr>
<tr>
<td>Matrix total compressibility, $c_m$, psi$^{-1}$</td>
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</tr>
<tr>
<td>Natural fracture permeability, $k_{nf}$, md</td>
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<td></td>
</tr>
<tr>
<td>Natural fracture porosity, $\Phi_{nf}$</td>
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<td></td>
</tr>
<tr>
<td>Natural fracture total compressibility, $c_{nf}$, psi$^{-1}$</td>
<td>2.5E-04</td>
<td></td>
</tr>
<tr>
<td>Hydraulic fracture permeability, $k_{hf}$, md</td>
<td>1E05</td>
<td></td>
</tr>
<tr>
<td>Hydraulic fracture porosity, $\Phi_{hf}$</td>
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<td></td>
</tr>
<tr>
<td>Natural fracture total compressibility, $c_{hf}$, psi$^{-1}$</td>
<td>2.5E-04</td>
<td></td>
</tr>
<tr>
<td>Hydraulic fracture total compressibility, $c_{hf}$, psi$^{-1}$</td>
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<td></td>
</tr>
<tr>
<td>Initial reservoir pressure, $P_i$, psi</td>
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<td></td>
</tr>
<tr>
<td>Constant flowing bottomhole pressure, $P_{wfr}$, psi</td>
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<tr>
<td>Reservoir temperature, $T$, °F</td>
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</tr>
<tr>
<td>Klinkenberg coefficient, psi</td>
<td>200</td>
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<td>Non-Darcy flow constant, $C_g$, m$^{3/2}$</td>
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<tr>
<td>Langmuir’s pressure, $P_L$, psi</td>
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<tr>
<td>Langmuir’s volume, $V_L$, scf/ton</td>
<td>218.57</td>
<td></td>
</tr>
<tr>
<td>Natural fracture total compressibility, $c_{nf}$, psi$^{-1}$</td>
<td>2.5E-04</td>
<td></td>
</tr>
</tbody>
</table>

In this single porosity model, the formation has no natural fractures with homogeneous and low-permeability, matrix and a horizontal well at the center. A refining grid is built for the simulation. The basic parameter set for the simulation is summarized in Table 2. In this system, fluids include gas and water, but water is at residual or immobile, so it is a single-phase gas flow problem. To simulate transient gas flow of this system using our model, the hydraulic fracture is represented by a discrete fracture with infinite conductivity. Using the parameters in the table, the dimensionless fracture conductivity is calculated as $C_{fd} = k_f w_f / k_m x_f = 500$ and the hydraulic fracture could be treated as infinite conductivity.
the same, to early transient gas flow through extremely low permeability rock.

Next we analysed the sensitivity of fracture-matrix total contact area for the transient pressure behavior in earlytime. Several models with different area are built and Fig. 9 is the simulation result. Larger dimensionless surface area will lead to a slower increase of the dimensionless pseudo pressure. Slope of this line is inversely proportional to the fracture area.
For the gas flow problem with horizontal well, hydraulic fractures and SRV, we compared the natural fractured reservoir and the single porosity reservoir. Fig. 10 and Fig. 11 showed the comparison of pressure behavior between these two formations. It should be mentioned that flow does not appear as a straight on the plot of normalized pressure vs. the square root of time for double-porosity formation. This is because formation transmissibility is so high in this situation that linear flow period will not last long.

Gas adsorption influence is also analysed in Fig. 12 and Fig. 13 at two initial pressures: one is 2350 psi and the other is 3800 psi. Two conclusions could be got from these two figures:

1. Gas flow with adsorption will also behave straightly in the normalized pressure vs. the square root of time plot for the linear flow period. This is identical with our previous analysis that adsorption could be treated by a compressibility factor if the pressure changes a little.

2. Adsorption will have different influences on the linear flow behavior at different initial pressure, as shown in Fig. 2 that “adsorption compressibility” drops with pressure increases.
Conclusion
In this paper, we use a numerical model to simulate shale gas production with a continuum, discrete or hybrid modeling approach. Our modeling studies indicate that the most sensitive parameter of hydraulic fractures to early transient gas flow responses through extremely low permeability rock is actually the fracture-matrix contacting area, generated by fracturing stimulation. Based on this observation, we demonstrate that it is possible to use transient pressure testing data to estimate the area of fractures generated from fracturing operations. A methodology using typical transient pressure responses, simulated by the numerical model, to estimate fracture areas created or to quantity hydraulic fractures with traditional well testing technology is presented. The methodology as well as type curves of pressure transients from this study can be used for quantify hydraulic fractures in field application.

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References
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