A Practical Compositional Method for Simulation of CO₂ Flooding in Porous and Fractured Petroleum Reservoirs

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Abstract

CO₂ flooding is one of the most effective and used methods for enhanced oil recovery (EOR) approaches. The number of CO₂ flooding projects has increased rapidly in China and around the world. Compositional simulation is required for evaluating CO₂ flooding in EOR operations, especially for miscible or nearly miscible flooding when black-oil simulation is no longer adequate. The simulation method proposed here is a multi-dimensional, three-phase, and compositional modeling approach, which is applicable to both porous and fractured reservoirs. In the model formulation, a generalized multi-continuum approach is adopted to handle flow and transport in naturally fractured reservoirs and the mass flux of each mass component is contributed by advection and diffusion processes. In addition, precipitation of heavy oil components and absorption of CO₂ on the solid grains are modeled based on reversible linear or nonlinear isotherms. The governing partial differential equations for conservation of each component are discretized using a finite volume method and the resulting discrete equations are solved fully implicitly by Newton-Raphson iteration. The equation of state (EOS) by Soave-Redlich-Kwong is used to calculate the physical properties of fluids. Research has shown that the flash calculations with EOS in compositional simulation are computationally intensive and may not be reliable at near critical conditions. Therefore, a K-value based approach, improved by Almheidaeb et al. (2002), is used for partitioning of oil components and CO₂ between oil and CO₂ phases. In addition, the laboratory measured oil and CO₂ phase compositional data can be used alternatively to account for compositional effect in this model. Two numerical examples are presented to show that the proposed modeling method is efficient for simulation of CO₂ flooding processes in EOR operations.

Introduction

CO₂ flooding has been used as a commercial process for enhanced oil recovery (EOR) for over 40 years and is the second most applied EOR process in the world (Jarrell et al., 2002). Both laboratory studies and field applications have established that CO₂ can be an efficient oil-displacing agent. CO₂ injection in mature hydrocarbon fields has also been considered as a favourable option to reduce accumulation of atmospheric CO₂ and thus mitigate greenhouse effects on climate (Bradshaw and Cook, 2001). As the results, a number of CO₂ flooding projects has increased rapidly in China and around the world. Numerical simulation is the most common technique in the oil industry to better understand, predict, design, and manage a CO₂-EOR project. Miscibility between oil and CO₂ will occur when pressure is high enough to compress the CO₂ to a density at which it becomes a good solvent for the lighter hydrocarbons in the crude oil. Compositional simulation is required for modeling the complex interaction of flow in CO₂-EOR operations, especially for miscible or nearly miscible flooding when black-oil simulation is no longer adequate. In recent years, we have seen more CO₂ application in naturally fractured reservoirs. Due to the large permeability contrast between the matrix and the fracture in fractured reservoirs, injected fluids such as CO₂ or water easily move toward the production wells and result in bypassing of the matrix oil and poor sweep efficiency. Early gas or water breakthrough becomes problematic in the secondary oil recovery stage in most fractured reservoirs.

Among the commonly used conceptual models for analyzing flow through fractured rock, dual-continuum models, i.e., double- and multi-porosity, and dual-permeability, are perhaps the most popular approaches used in fractured reservoir modeling studies (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969). A more general and efficient method for multi-continuum, named multiple-interacting-continua (MINC) method, was proposed (Pruess and Narasimhan, 1985; Wu and Pruess, 1988). The TOUGH2 family of multiphase flow numerical simulators (Pruess, 2004) has a long recognized record of
applications in different fields of mass and heat transport in porous media and fractured reservoirs. Within the TOUGH2 family, TMVOC (Pruess and Battistelli, 2002) is a compositional simulator that was developed to model the migration of three-phase multi-component inorganic gases and hydrocarbons mixtures for environmental applications. However, because Henry's law and Raoult's law were adopted for phase equilibrium calculations, TMVOC is not applicable for conventional hydrocarbon reservoirs.

Phase equilibrium calculation is important for the transfer or partitioning of species between the phases to account for the effect of phase composition change on fluid properties. There are two methods to calculate equilibrium compositions and phase molar fractions in compositional simulation: K-value approach and equation-of-state (EOS) based on flash calculation. The most common way in compositional simulation was to perform a two-phase flash calculation. Rachford and Rice derived a simple objective function, Rachford-Rice function, to determine the phase mole fractions and phase compositions for a fixed overall composition and set of specified K-values (Rachford and Rice, 1952). The Rachford-Rice method has been extended by Li and Nghiem (1982) to negative flash calculations, where the overall composition can be outside the two-phase zone. Whitson and Michelsen (1989) proposed a more robust method by specifying a window in which the correction root of the objective function should lie. Although there have been significant progress and advance in two-phase equilibrium calculations with EOS, flash calculations in compositional simulation are computational intensive and may not be reliable near critical regions. Three-phase equilibrium calculations for CO$_2$-water-hydrocarbon mixtures are required in compositional reservoir simulation of CO$_2$ flooding modeling. However there is a lack in progress for three-phase flash calculation in compositional reservoir simulation despite a large number of publications on the subject (Haugen et al., 2007). The compositional reservoir simulations are computationally very expensive primarily due to three-phase flash calculations, leading to the use of small number of grid blocks and high numerical dispersion, hence full-field compositional simulation usually is not possible.

Although the K-value approach may not automatically satisfies the thermodynamic consistency requirements, the compositional simulation based on K-value approach is robust and efficient, and the parameters for phase equilibrium calculations are directly related to the experimental data of crude oil. Wilson (1968) proposed a general correlation for K-values from experimental data. This simplified expression was modified by Whitson and Torp (1983) to accommodate the compositional effects at high pressures, K-value charts and the according polynomial equation for light hydrocarbons vs. pressure and temperatures that are valid up to around 6,000 psi pressures or more, are presented by DePriester (1953) and McWilliams (1973), respectively. Based on their work and experimental results, a new correlation for computing K-values for high-pressure systems was proposed by Almehaideb et al. (2002).

The main objective of this work is to propose a robust, efficient, compositional and implicit method for numerical simulation of CO$_2$ flooding in multiphase, three dimensional, porous and fractured reservoirs. In this compositional simulation method, the K-value correlation improved by Almehaideb et al. (2002), is used for partitioning of CO$_2$ and oil components in CO$_2$-water-hydrocarbon mixtures. The multiphase fluid flow is described by Darcy equation, and the diffusion and mechanical dispersion of multicomponent are described by an extended model based on Fick’s law. Adsorption of CO$_2$ on the rock grains, and precipitation of heavy component are considered to obey a linear or nonlinear isotherm. A residual based Newton-Raphson algorithm is used to solve the governing equations.

Mathematical Model
It is assumed that the multicomponent fluids are in local chemical and thermal equilibrium, and no chemical reactions take place. Multiphase fluid flow, multicomponent transport, and heat transfer through porous and fractured media are governed by the fundamental conservation laws mass, momentum, and energy. The model formulations are presented in this section.

Multiphase Flow
Darcy’s law proposed originally for single-phase flow has been extended to describe the flow of multiple, immiscible fluids (Scheidegger, 1974). The generalized Darcy’s law for the simultaneous flow of immiscible fluids in a multiphase system is given as

$$ \mathbf{v}_\beta = -\frac{k_{\beta \beta}}{\mu_\beta} \left( \nabla P_\beta - \rho_\beta g \nabla z \right) $$

where $\mathbf{v}_\beta$ is a vector of the Darcy’s velocity, or volumetric flow rate, of phase $\beta$, with $\beta$ being fluid phase index ($\beta = g$ for CO$_2$; $\beta = w$ for water; and $\beta = n$ for oil); $P_\beta$, $\mu_\beta$, $\rho_\beta$, and $g$ are pressure, viscosity and density (all mass densities in this paper are expressed in molar units) of fluid phase $\beta$, and gravitational constant, respectively; $z$ is the vertical coordinate; $k$ is absolute or intrinsic permeability; and $k_{\beta \beta}$ is the relative permeability to fluid phase $\beta$.

Transport of Diffusion and Mechanical Dispersion
The combined effects of diffusion and mechanical dispersion are often called hydrodynamic dispersion and are described using the Fick’s law for transport through a single-phase porous medium (Scheidegger, 1961; Bear, 1972). Fick’s law has been generalized to describe the transport of components in multiphase, miscible, or immiscible fluid systems (Corapcioglu and
Baehr, 1987; Sleep and Sykes, 1989). The generalized Fick’s law for hydrodynamic dispersion effects in a multiphase system is used to evaluate dispersive flux of transport.

\[ \mathbf{f}_\beta^k = -D_{\beta}^k \nabla (\rho_\beta x^k_\beta) \]  

(2)

where superscript \( k \) is an index for mass components (for \( \text{CO}_2 \), water, oil); \( \mathbf{f}_\beta^k \) is the dispersive mass flux vector of component \( k \) within fluid \( \beta \); \( x^k_\beta \) is the mole fraction of component \( k \) in fluid \( \beta \); and \( D_{\beta}^k \) is the hydrodynamic dispersion tensor accounting for both molecular diffusion and mechanical dispersion for component \( k \) in phase \( \beta \).

The general dispersion model (Scheidegger, 1961) was improved to include multiphase fluid effects as (Wu and Pruess, 2000),

\[ D_{\beta}^k = \alpha_\beta^T \mathbf{v}_\beta | \delta_{ij} + (\alpha_\beta^L - \alpha_\beta^T) \frac{\mathbf{v}_\beta \cdot \mathbf{v}_\beta}{| \mathbf{v}_\beta |} + \phi S_\beta \tau d^k_\beta \delta_{ij} \]  

(3)

where \( \alpha_\beta^T \) and \( \alpha_\beta^L \) are transverse and longitudinal dispersivities, respectively, in fluid \( \beta \) of porous media; \( \phi \) is the effective porosity of the porous medium; \( S_\beta \) is saturation of fluid \( \beta \); \( \tau \) is tortuosity of the porous medium; \( d^k_\beta \) is the molecular diffusion coefficient of component \( k \) within fluid \( \beta \); and \( \delta_{ij} \) is the Kronecker delta function (\( \delta_{ij} = 1 \) for \( i = j \), and \( \delta_{ij} = 0 \) for \( i \neq j \)) with \( i \) and \( j \) being coordinate indices.

**Heat Transfer**

Heat transfer in porous media is in general a result of both convective and conductive processes. Heat convection is contributed by thermal energy carried mainly by bulk flow of all fluids as well as by dispersive mass fluxes. On the other hand, heat conduction is driven by temperature gradients and may follow Fourier’s law. Then the overall heat flux vector may be described as,

\[ \mathbf{F}^T = \sum_\beta \left( h_\beta \rho_\beta \mathbf{v}_\beta \right) - \sum_\beta \sum_k \left( h^k_\beta D_{\beta}^k \cdot \nabla (\rho_\beta x^k_\beta) \right) - (K_T \nabla T) \]  

(4)

where \( \mathbf{F}^T \) is the combined heat flux vector, including both advective and conductive heat flow in a multiphase, multicomponent system; \( h_\beta \) and \( h^k_\beta \) are specific enthalpies of fluid phase \( \beta \) and of component \( k \) in fluid \( \beta \), respectively; \( K_T \) is the overall thermal conductivity, which is a function of fluid saturations; and \( T \) is temperature.

**Absorption of \( \text{CO}_2 \) and Precipitation of Heavy Oil Components**

Adsorption of \( \text{CO}_2 \) component onto solid grains is a kinetic process. If the rate of adsorption and desorption is fast relative to other processes occurring in the reservoirs, adsorption of \( \text{CO}_2 \) can be modeled as reversible instantaneous sorption isotherms.

The equilibrium adsorption may be related to the concentration of component \( k \) (\( \text{CO}_2 \)) in phase \( \beta \) as,

\[ M_{\text{ads}}^k = (1 - \phi) \rho_s K_{d,\beta}^k x^k_\beta \]  

(5)

where \( M_{\text{ads}}^k \) is the mass of component \( k \) adsorbed per unit reservoir volume; \( \rho_s \) is the density of rock solids; \( K_{d,\beta}^k \) is the distribution coefficient of component \( k \) between the phase \( \beta \) and rock solids to account for adsorption effects, it is treated as a constant for the linear isotherm, or as a function of the concentration or mole fraction in a fluid phase and a maximum sorptive capacity when the non-linear Langmuir isotherms are considered.

Precipitation of heavy oil components from reservoir fluids during \( \text{CO}_2 \) injection is a serious problem, because it can plug the formation, cause permeability reduction and alteration of rock wettability from water wet to oil wet. In order to avoid four-phase equilibrium calculations, it is assumed that precipitation of heavy oil components on the solid grains can be modeled as the absorption isotherms (See Eq. (5)), whereas the distribution coefficient should be also a function of mole fraction of \( \text{CO}_2 \) in the fluid phase.

**Governing Equations**

The multiphase system consists of three fluid phases, \( \text{CO}_2 \), water, and oil, and each fluid in turn consists of a number of mass components. A combination of mass and energy conservation principle with Eqs. (1-4) gives rise to a set of governing equations described below. The mass conservation equations of each component \( k \) in the porous continuum can be written as follows:
\[
\frac{\partial}{\partial t} \left\{ \phi \sum_{\beta} \left( \rho_\beta S_\beta x_\beta^k + (1 - \phi) \sum_{\beta} \rho_\beta x_\beta^k K_{d,\beta} \right) \right\} + \lambda_k \left\{ \phi \sum_{\beta} \left( \rho_\beta S_\beta x_\beta^k + (1 - \phi) \sum_{\beta} \rho_\beta x_\beta^k K_{d,\beta} \right) \right\} \\
- q^k - R^k = -\sum_{\beta} \nabla \cdot \left( \rho_\beta x_\beta^k \mathbf{v}_\beta \right) + \sum_{\beta} \nabla \cdot \left( \mathbf{D} \cdot \nabla \left( \rho_\beta x_\beta^k \right) \right)
\]

and the energy conservation equation is:

\[
\frac{\partial}{\partial t} \left\{ \sum_{\beta} \left( \phi \rho_\beta S_\beta U_\beta \right) + (1 - \phi) \rho_s U_s \right\} - q^T - R^T = -\sum_{\beta} \nabla \cdot \left( \mathbf{h}_\beta \rho_\beta \mathbf{v}_\beta \right) \\
+ \sum_{\beta} \nabla \cdot \left( h_\beta \mathbf{D} \cdot \nabla \left( \rho_\beta x_\beta^k \right) \right) + \nabla \cdot \left( \mathbf{K}_f \nabla T \right)
\]

where \( k \) is the index for the components, \( k = 1, 2, 3, \ldots, N_c \), with \( N_c \) being the total number of mass components; \( \rho_s \) is the density of rock solids; \( \lambda_k \) is the radioactive decay constant of component \( k \); \( \mathbf{D} \) is the hydrodynamic dispersion tensor; \( q^k \) and \( q^E \) are external source/sink terms or fracture-matrix exchange terms for component \( k \) and energy; \( R^k \) and \( R^T \) is the internal generation term for component \( k \) and for energy, respectively; and \( U_\beta \) and \( U_s \) are the internal energies of fluid \( \beta \) and rock solids, respectively.

### Constitutive Relationships and Fluid Properties

For a flow system consisting of \( N_c \) components and \( N_p \) phases, the total number of primary thermodynamic variables is \( N_c + 1 \). The constitutive correlations express inter-relations and constraints of physical processes, variables, and parameters, and allow the evaluation of thermophysical properties as functions of the primary unknown variables.

### Saturation and Mass Fraction Constraints

For fluid saturations,

\[
\sum_{\beta} S_\beta = 1
\]

(8)

The mass fractions of component \( k \) within phase \( \beta \) are subject to

\[
\sum_k x_{\beta}^k = 1
\]

(9)

### Capillary Pressure and Permeability

In a multiphase system, the capillary pressure relates pressures between the phases. When dealing with a three-phase system of gas (CO₂), water, and oil, water- and gas-phase pressures are related by

\[
P_w = P_g - P_{cgw} \left( S_w, S_g \right)
\]

(10)

where \( P_{cgw} \) is the gas-water capillary pressure. The oil phase pressure is related to the gas phase pressure through

\[
P_n = P_g - P_{cgw} \left( S_w, S_n \right)
\]

(11)

where \( P_{cgw} \) is the gas-oil capillary pressure in a three-phase system.

The oil-water capillary pressure, \( P_{cnw} \), in a three-phase system can be derived from,

\[
P_{cnw} = P_{cgw} - P_{cnw} = P_n - P_w
\]

(12)

The relative permeabilities of a fluid phase in a three-phase system are normally assumed to be functions of fluid saturation only as

\[
k_{r\beta} = k_{r\beta} \left( S_w, S_g \right)
\]

(13)

The functions of capillary pressure and relative permeability are generally determined from laboratory and field studies for a given site in a tabulated form.
From the mass balances, the precipitation of heavy oil components is obtained. The volume fraction of precipitated heavy oil components in the original pore volume can be calculated, the remaining void space is available for fluid flow. The impact of porosity change on formation permeability depends not only on the overall reduction of porosity but on details of the pore space geometry and the distribution of precipitate within the pore space. Several models of permeability reduction can be selected (Pruess et al., 1999).

**Properties of Gas Phase**

Density of CO\textsubscript{2} as a function of pressure at various temperatures (Hendriks and Blok, 1993) and viscosity of CO\textsubscript{2} as a function of pressure at various temperatures (Goodrich, 1980) are adopted in this paper.

The Soave-Redlich-Kwong (SRK) equation (Redlich and Kwong, 1949; Soave, 1972) of state is used to describe the PVT properties of gas phase. For a pure component the cubic equation may be written in the form

\[ Z^3 - Z^2 + (A^* - B^* - (B^*)^2)Z - A^*B^* = 0 \]  

(14)

where the largest root \( Z \) is the gas compressibility factor; \( A^* = \frac{aP_c}{RT_c^2} \) and \( B^* = \frac{bP_c}{RT_c} \); \( a \) and \( b \) are parameters that depend on critical pressure \( P_c \), critical temperature \( T_c \) and Pitzer’s acentric factor \( \omega \); with \( R \) the universal gas constant.

\[ a = \frac{0.42748RT_c^2}{P_c}[1 + (0.48 + 1.574\omega - 0.176\omega^2)(1 - [T/T_c^{1/2}]^2)]^2 \]  

(15)

\[ b = \frac{0.08664RT_c^2}{P_c} \]  

(16)

For a multicomponent mixture, if the mixing rules recommended by Reid et al. (1987) are adopted, the two parameters are written as,

\[ a_m = \left( \sum_k x_k^a \sqrt{a_k} \right)^2 \]  

(17)

\[ b_m = \sum_k x_k^ab_k \]  

(18)

The molar density of the gas phase is given by

\[ \rho_g = P(\text{RTZ}_g) \]  

(19)

Using the Chapman-Enskog kinetic theory, gas phase viscosity \( \mu_g \) is obtained as a function of pure component viscosities \( \mu_k \).

\[ \mu_g = \sum_{k=1}^{N} \sum_{k=1}^{N} x_k^a \mu_k \Phi_{k\lambda} \]  

(20)

The \( \Phi_{k\lambda} \) are binary interaction parameters given by

\[ \Phi_{k\lambda} = \frac{(1 + \sqrt{\mu_k/\mu_\lambda} \sqrt{M_\lambda/M_k})^2}{\sqrt{8(1 + M_\lambda/M_k)}} \]  

(21)

where \( M_k \) is the molecular weight of component \( k \).

**Properties of Oil Phase**

The molar density of the oil phase is given by

\[ \rho_o = P(\text{RTZ}_o) \]  

(22)

where \( Z_o \) is the compressibility factor of oil phase, and can be calculated from Eq. (14) for hydrocarbon mixture.
The viscosity of a pure hydrocarbon component is calculated as a function of temperature from an equation due to Yaws et al. (1976).

\[ \ln \mu = A - \frac{B}{T} + C'T + D'T^2 \]  

(23)

where \( A'B'C'D \) are the constants, and their values can be found in Ref. by Reid et al. (1987).

The mixture viscosity of the oil phase is obtained approximately in terms of the pure component viscosities as

\[ \mu_m = \prod_{k=1}^{N_c} \mu_k^{x_k} \]  

(24)

Properties of Water Phase

Effects on thermodynamic properties of water phase from dissolution of hydrocarbon and CO\(_2\) are neglected. Therefore density and viscosity of the water phase are taken to depend only on pressure and temperature, and can be calculated by the steam table equations issued by the International Formulation Committee (IFC, 1967).

Phase Partitioning and K-Value approach

In a multicomponent system, the equilibrium ratio, \( K_{\alpha\beta}^k \), of a given component \( k \) is defined as the ratio of the mole fraction of the component in the phase \( \alpha \) to the mole fraction of the component in the phase \( \beta \).

\[ K_{\alpha\beta}^k = \frac{x_k^\alpha}{x_k^\beta} \]  

(25)

For the water-oil equilibrium, the solubility of water in oil phase and the dissolution of hydrocarbon components in water phase are generally small. The equilibrium constants for water-oil partitioning are

\[ K_{\text{w-o}}^n = \frac{x_{w,\text{sol}}^n}{x_{\text{o},\text{sol}}^n} \]  

(26)

\[ K_{\text{w-o}}^w = \frac{1}{x_{\text{o},\text{sol}}^w} \]  

(27)

where the superscripts \( n \) and \( w \) denote hydrocarbon components and water, respectively. \( x_{w,\text{sol}}^n \) is the solubility of hydrocarbon component in water phase and \( x_{\text{o},\text{sol}}^w \) is the solubility of water in oil phase. The solubility is an increasing or decreasing function of temperature (Adenekan et al., 1993).

For the gas-water equilibrium, dissolution of CO\(_2\) in water phase can be calculated by Henry's law and the gas mixture is assumed to obey Dalton's law.

\[ K_{\text{g-w}}^{\text{CO}_2} = \frac{K_H}{P_g} \]  

(28)

\[ K_{\text{g-w}}^{\text{H}_2\text{O}} = \frac{P_{\text{sup}}^{\text{H}_2\text{O}}}{P_g} \]  

(29)

where \( K_H \) is the Henry's coefficient for CO\(_2\) dissolution in water phase, and can be computed from a polynomial fit to experimental data as a function of temperature (Battistelli et al., 1997). \( P_{\text{sup}}^{\text{H}_2\text{O}} \) is the saturated vapor pressure of water at the given temperature.

For the gas-oil equilibrium, the K-value correlation improved by Almehaideb et al. (2002) is adopted.

\[ K_{\text{g-o}}^i = \left( \frac{P_{ci}}{P_k} \right)^{A-1} \left( \frac{P_{ci}}{P} \right)^{\exp(AK_i^*)} \]  

(30)
where the superscripts \( i \) denote hydrocarbon components or \( \text{CO}_2 \). \( p \) is the system pressure (psia), \( p_c \) is the convergence pressure (psia), \( p_{ci} \) is the critical pressure of component \( i \) (psia), \( T \) is the system temperature (°R), \( \omega \) is the acentric factor, \( a_{T1}, a_{T2}, a_{T3}, a_{p1}, a_{p2}, a_{p3}, a_{\omega} \) are constants for component \( i \) from experimental data.

**Numerical Algorithm and Numerical Examples**

The governing equations (Eqs. 6 and 7) are solved using a fully implicit method. A kind of finite volume method, the integral finite difference method (Narasimhan and Witherspoon, 1976; Pruess, 1991) is used as the spatial discretization scheme. The integral finite difference method has the flexibility of finite element method and no predetermined limit on the number of neighbors that a grid block can have. The time discretization is carried out using a backward, first-order finite-difference scheme. Finally, the discretized formulae of component mass and energy balance equations are solved iteratively by Newton-Raphson method.

Depending on pressure, temperature and relative abundance of the different components, the fluids may exist in three different phase combinations, each of the phase may appear and disappear in a grid block during the simulation. The phases of each grid block are checked for the thermodynamic compatibility with the new values of the primary variables at the end of every Newton-Raphson iteration. The numerical algorithm for the treatment of phase appearance and disappearance (Adenekan et al., 1993) is adopted, the primary variables in the grid block are switched accordingly when there is a phase appear or disappear.

A compositional simulator is developed on the basis of the method proposed in this paper. Two numerical examples are simulated. The first one is a one-dimensional homogeneous isothermal geological model with one injection well of \( \text{CO}_2 \) and one production well of oil, as shown in Fig. 1. The fluid mixture has five pseudo-oil components and the fluids properties are listed in Table 1. The injection rate is constant and the production well bottomhole pressure is maintained at the initial pressure of the model. The relative permeability curves are shown in Fig. 2. The capillary pressures are assumed to be negligible and the input data are summarized in Table 2. The oil recoveries obtained from the simulation results with different bottomhole pressures are shown in Fig. 3. It can be seen that the simulated minimum miscibility pressure is of the order of \( 21.8 \text{MPa} \).

In the second example, the \( \text{CO}_2 \) flooding in a two dimensional cross sectional synthetic model is simulated. The reservoir is considered to be a dual-continuum system. The grid is shown in Fig. 4 with \( 50 \times 1 \times 10 \) grid blocks and initially saturated by oil. The fluids properties and the relative permeability are the same as that of the first example, and the capillary pressures are also assumed to be negligible. Boundaries of the model domain are no-flow. The initial pressure is \( 20 \text{MPa} \). The injected fluid is \( \text{CO}_2 \) at a rate of \( 3. \text{Im}^3 \text{/ day} \), located at Grid (1, 1). The producing well is located at Grid (1, 50) and the production pressure is \( 10 \text{MPa} \). The summarized input data are given in Table 3. The distributions of oil saturation in matrix and fracture continua after \( \text{CO}_2 \) injection of 60 days are shown in Fig. 5 and Fig. 6, respectively. The result shows that the injection \( \text{CO}_2 \) fluids easily move toward the production well and result in bypassing of the matrix oil and poor sweep efficiency. This is mainly due to the large permeability contrast between the matrix and the fracture in this model.

**Conclusions**

A three-dimensional, three-phase compositional modeling method for simulation of \( \text{CO}_2 \) flooding has been implemented and tested. Because the proposed method is based on a generalized multi-continuum approach, it is applicable to both porous and fractured reservoirs. The multiphase fluid flow is described by Darcy equation, the diffusion and mechanical dispersion of multicomponent are described by an extended Fick’s law. Adsorption of \( \text{CO}_2 \) on the rock grains, and precipitation of heavy component are considered to obey a linear or nonlinear isotherm. The governing partial differential equations for conservation of each component are discretized using a finite volume method and the resulting discrete equations are solved fully implicitly by Newton-Raphson iteration. The K-value approach improved by Almehaideb et al. (2002), is used for partitioning of oil components and \( \text{CO}_2 \) between oil and gas phases. The simulated minimum miscibility pressure is presented by a one-dimensional homogeneous example. The numerical results of a two-dimensional dual-continuum example shows that injection \( \text{CO}_2 \) fluids can easily move toward the production well in the fractured reservoirs, early gas breakthrough occurs and results in bypassing of the matrix oil and poor sweep efficiency.
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References


Table 1. Initial composition and oil properties

<table>
<thead>
<tr>
<th>Component name</th>
<th>$p_c (psia)$</th>
<th>$T_c (°F)$</th>
<th>$\omega$</th>
<th>$a_{T1}$</th>
<th>$a_{T2}$</th>
<th>$a_{T3}$</th>
<th>$a_{p1}$</th>
<th>$a_{p2}$</th>
<th>$a_{p3}$</th>
<th>$a_{m}$</th>
<th>Mole fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>1071.0</td>
<td>547.90</td>
<td>0.2250</td>
<td>-292859</td>
<td>-0.01414</td>
<td>16.2443</td>
<td>-0.564467</td>
<td>48.8616</td>
<td>-1311.41</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Comp-1</td>
<td>667.8</td>
<td>343.37</td>
<td>0.0104</td>
<td>-292865</td>
<td>-0.00518</td>
<td>5.92040</td>
<td>0.009982</td>
<td>35.2435</td>
<td>54.8676</td>
<td>0.0</td>
<td>0.18</td>
</tr>
<tr>
<td>Comp-2</td>
<td>707.8</td>
<td>550.09</td>
<td>0.0986</td>
<td>-970689</td>
<td>0.0</td>
<td>-3.56546</td>
<td>0.622591</td>
<td>5.0297</td>
<td>688.0477</td>
<td>0.0</td>
<td>0.18</td>
</tr>
<tr>
<td>Comp-3</td>
<td>616.3</td>
<td>666.01</td>
<td>0.1524</td>
<td>-1481582</td>
<td>0.0</td>
<td>-6.38713</td>
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<td>-3.39020</td>
<td>747.110</td>
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<td>Comp-4</td>
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<td>829.10</td>
<td>0.2223</td>
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<td>-8.45991</td>
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<td>873.201</td>
<td>0.0</td>
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<tr>
<td>Comp-5</td>
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<td>1139.40</td>
<td>0.5069</td>
<td>-1778901</td>
<td>0.01436</td>
<td>6.00502</td>
<td>-1.10606</td>
<td>-1.03540</td>
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<td>0.0</td>
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Table 2. Summary of input data for one-dimensional model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>$10 \times 1 \times 1$</td>
</tr>
<tr>
<td>Grid block size</td>
<td>$1m \times 1m \times 1m$</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>$90°C$</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.3</td>
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<tr>
<td>Permeability</td>
<td>16.0 md</td>
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</tbody>
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Table 3. Summary of input data for two-dimensional sectional model

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<td>Dimension</td>
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</tr>
<tr>
<td>Grid block size</td>
<td>$2m \times 1m \times 2m$</td>
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<tr>
<td>Initial temperature</td>
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<tr>
<td>Matrix porosity</td>
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<tr>
<td>Fracture porosity</td>
<td>0.05</td>
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<tr>
<td>Matrix permeability</td>
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<tr>
<td>Fracture permeability</td>
<td>1600 md</td>
</tr>
</tbody>
</table>

Fig. 1. A one-dimensional homogeneous isothermal geological model
Fig. 2. The relative permeability curves of oil phase and gas phase

Fig. 3. Simulation results of the one-dimensional homogeneous model with CO₂ as injection gas

Fig. 4. Grid diagram for two-dimensional vertical section model

Fig. 5. Oil saturation for fracture continua of the two-dimensional vertical section model after CO₂ injection of 60 days

Fig. 6. Oil saturation for matrix continua of the two-dimensional vertical section model after CO₂ injection of 60 days