Abstract

A multicontinuum conceptual model is presented and implemented into a three-dimensional, three-phase reservoir simulator, using a generalized multicontinuum modeling approach. The conceptual model, proposed for investigating multiphase flow and displacement through naturally fractured vuggy carbonate reservoirs, is based on observation and analysis of geological data, as well as on core examples from the carbonate Tahe Oil Field in China. In this conceptual model, naturally fractured vuggy rock is considered to be a triple-continuum medium, consisting of (1) highly permeable and well-connected large-scale fractures; (2) low or impermeable rock matrix; and (3) various-sized vugs or cavities. The base matrix system may contain many small or isolated cavities (of centimeters or millimeters in diameter), and large cavities (or vugs) ranging from centimeters to meters in diameter. Vugs may be (1) directly connected to large fractures, (2) indirectly connected to large fractures through small fractures or microfractures, or (3) isolated from large fractures by rock matrix. Similar to the conventional double-porosity concept, the fracture continuum is responsible for the occurrence of global flow, whereas vuggy and matrix continua (mainly providing large-storage space of source/sink) are locally connected to each other as well as interacting with globally connected fractures. In the numerical implementation, a control-volume, integral finite-difference method is used for spatial discretization, and the resulting discrete nonlinear equations for the three-phase fluids, coupled with each continuum, are solved fully implicitly by Newton iteration. The numerical scheme, verified by comparing its results against those of available analytical solutions, is used to simulate water-oil flow through the fractured vuggy reservoirs of Tahe.

Introduction

Naturally fractured reservoirs existing throughout the world represent a significant amount of the world oil and gas reserves. Since the 1960s, studies of flow and transport in fractured rock have received increasing attention, and significant progress has been made in numerical modeling of flow and transport processes in fractured reservoirs. Research efforts, driven by the increasing need to develop petroleum and geothermal reservoirs (as well as to resolve subsurface contamination problems), have developed many numerical modeling approaches and techniques.

In the past decade, the petroleum industry has faced a growing demand for oil and natural gas, while at the same time few new oil reservoirs have been found worldwide. The efficient development of naturally fractured reservoirs has become a top priority of the oil industry. Because of the known low oil recovery rates (in general) from naturally fractured reservoirs, interest in enhancing oil and gas recovery from such reservoirs has grown, with more investigations conducted for multiphase flow and transport phenomena in fractured reservoirs. Since the 1970s, in parallel to the development in the oil industry, environmental concerns over subsurface contamination have motivated many studies of fluid, chemical, and heat transport in variously saturated fractured formations. Moreover, suitability evaluations for underground geological storage of high-level radioactive waste in fractured rock have generated renewed interest in investigations of multiphase and radionuclide transport in a fractured geological system.

Even though significant progress has been made towards the understanding and modeling of flow and transport processes in fractured rock since the 1960s, most of those studies have focused on naturally fractured reservoirs, without including cavities. Recently, driven by the need to develop underground natural resources, and by environmental concerns, characterizing vuggy fractured rock has currently received attention, because many naturally fractured vuggy reservoirs have been found worldwide and can significantly contribute to reserves of oil and gas. Significant interest is being generated in investigating vuggy fractured reservoirs.

Mathematical approaches to modeling flow through fractured reservoirs in general rely on continuum approaches and involve developing conceptual models, incorporating the geometrical information of a given fracture-matrix system, setting up mass and energy conservation equations for fracture-matrix domains, and then solving discrete nonlinear algebraic equations. The commonly used mathematical methods for modeling flow through fractured rock include: (1) an explicit discrete-fracture and matrix model, (2) the dual...
continuum method, including double- and multiporosity, dual-permeability, or the more general "multiple interacting continuum" (MINC) method \(^{21, 12, 17}\), and (3) the effective-continuum method (ECM) \(^{23}\). Among these three approaches, the dual-continuum method has been perhaps the most models used in application. This is because it is computationally less demanding than the discrete-fracture approach, and it can handle fracture-matrix interactions under multiphase flow, heat transfer, and chemical transport conditions in fractured reservoirs.

In addition to the traditional double-porosity concept, a number of triple-porosity or triple-continuum models have been proposed \(^{8, 27, 1, 4, 22}\) for describing flow through fractured rocks. In particular, Liu et al. \(^{14}\) and Camacho-Velazquez et al. \(^{7}\) present several new triple-continuum models for single-phase flow in a fracture-matrix system that include cavities within the rock matrix (as an additional porous portion of the matrix). In general, these models have focused on handling the heterogeneity of the rock matrix or fractures, e.g., subdividing the rock matrix into two or more subdomains with different porous medium properties.

The objectives of this study are (1) to propose a triple-continuum conceptual model to include effects of differently sized vugs and cavities on multiphase flow processes in naturally fractured vuggy reservoirs; (2) to develop a methodology for numerically implementing the proposed model; (3) to verify the proposed model; and (4) to demonstrate the model application to a field simulation of fractured vuggy reservoirs. In particular, we discuss the issues and the physical rationale for how to represent vugs using a numerical approach.

Conceptual and Mathematical Model

As observed in the carbonate formation of the Tahe Oilfield in western China \(^{28}\), a typical fractured vuggy reservoir consists of a large and well-connected fractured, low-permeable rock matrix, as well as a large number of cavities or vugs. Those vugs and cavities are irregular in shape and vary in size from millimeter to meters in diameter. Many of the small-sized cavities appear to be isolated from fractures. In this paper, we use “cavities” for small caves (with sizes of centimeters or millimeters in diameter), whereas “vugs” represent larger cavities (with sizes from centimeters to meters in diameter). Several conceptual models for vugs are shown in Figures 1, 2, and 3: (1) vugs are indirectly connected to fractures through small fractures or microfractures; (2) vugs are isolated from fractures or separated from fractures by rock matrix; and (3) some vugs are directly connected to fractures and some are isolated. In reservoirs, there are actually many more vug varieties and spatial distributions than those shown in Figures 1, 2, and 3, through some could be approximated by the conceptual models, represented by those figures or their combinations. In no circumstance, however, is there a need for uniform size-distribution patterns for vugs and cavities in this study.

Similar to the conventional double-porosity concept \(^{21}\), large fractures, or a fracture continuum, are conceptualized to be main pathways for global flow, while vuggy and matrix continua, locally connected to each other, as well as directly or indirectly interacting with globally connecting fractures, generally provide storage space as sinks or sources. Note that vugs and cavities directly connected with fractures (e.g., Figure 3) are considered part of the fracture continuum. More specifically, we conceptualize the fractured-vug-matrix system as consisting of (1) “large” fractures (or fractures), globally connected on the model scale, (2) various-sized vugs, which are locally connected to fractures either through “small” fractures or through rock matrix, and (3) rock matrix, which may contain a number of cavities, locally connected to large fractures and/or vugs.

In principle, the proposed triple-continuum model is considered to be a natural extension of the generalized multi-continuum (MINC) approach \(^{17, 26, 22}\). In this approach, an “effective” porous medium is used to approximate fractures, vugs, or rock matrix, respectively, by considering the three continua to be spatially overlapping. The triple-continuum conceptual model assumes that approximate thermodynamic equilibrium exists locally within each of the three continua at all times. Based on this local equilibrium assumption, we can define thermodynamic variables, such as pressures, fluid saturation, concentration, and temperature, for each continuum. Note that the triple-continuum model is not limited to the orthogonal idealization of the fracture system or uniform size or distribution of vugs and cavities, as illustrated in Figures 1, 2, and 3. Irregular and stochastic distributions of fractures and cavities can be handled numerically, as long as the actual distribution patterns are known.

A multiphase isothermal system in fractured vuggy reservoirs is assumed to be composed of three phases: oil, gas, and water. For simplicity, these three components are assumed to be present only in their associated phases, with each phase flowing in response to pressure, gravitational, and capillary forces according to Darcy’s law. Therefore, three mass-balance equations fully describe the system in an arbitrary flow region of the porous, fractured, vuggy domain:

For gas flow,

\[
\frac{\partial}{\partial t} \left( \phi (S_o \bar{\rho}_g + S_g \rho_g) \right) = -\nabla \cdot (\bar{\rho}_g \bar{V}_o + \rho_g \bar{V}_g) + q_g \tag{1}
\]

For water flow,

\[
\frac{\partial}{\partial t} (\phi S_w \rho_w) = -\nabla \cdot (\rho_w \bar{V}_w) + q_w \tag{2}
\]

For oil flow,

\[
\frac{\partial}{\partial t} (\phi S_o \bar{\rho}_o) = -\nabla \cdot (\bar{\rho}_o \bar{V}_o) + q_o \tag{3}
\]

where the Darcy’s velocity of phase \( \beta \) (\( \beta = g \) for gas, = w for water, and = o for oil) is defined as

\[
\bar{V}_\beta = -\frac{k k_{\beta}}{\mu_\beta} (\nabla P_\beta - \rho_\beta g \nabla D) \tag{4}
\]

In Equations (1)-(4), \( \phi \) is the effective porosity of the medium; \( \rho_\beta \) is the density of phase \( \beta \) at reservoir conditions; \( \bar{\rho}_o \) is the density of oil, excluding dissolved gas, at reservoir conditions; \( \bar{\rho}_{dg} \) is the density of dissolved gas (dg) in oil phase at
reservoir conditions; \( \mu_\beta \) is the viscosity of phase \( \beta \); \( S_\beta \) is the saturation of phase \( \beta \); \( P_\beta \) is the pressure of phase \( \beta \); \( q_\beta \) is the sink/source term of component \( \beta \) per unit volume of the medium, representing mass exchange through injection/production wells or resulting from fracture-matrix-vug interactions; \( g \) is gravitational acceleration; \( k \) is absolute/intrinsic permeability (tensor) of the medium; \( k_{ij} \) is relative permeability to phase \( \beta \); and \( D \) is depth.

Equations (1), (2) and (3), governing mass balance for three-phase flow, need to be supplemented with constitutive equations that express all the secondary variables and parameters as functions of a set of key primary thermodynamic variables. The following relationships will be used to complete the description of multiphase flow through fractured porous media:

\[
S_w + S_o + S_g = 1
\]

In addition, capillary pressure and relative permeability relations are also needed for each continuum, which are normally expressed in terms of functions of fluid saturations. The densities of water, oil, and gas, as well as the viscosities of fluids, can in general be treated as functions of fluid pressures.

**Numerical Formulation**

The governing equations, as discussed above, for multiphase flow in fractured vuggy reservoirs have been implemented into a general-purpose, three-phase reservoir simulator, the MSFLOW code. As implemented numerically, Equations (1), (2), and (3) are discretized in space using an integral finite-difference or control-volume finite-element scheme for a porous-fractured-vuggy medium. Time discretization is carried out with a backward, first-order, finite-difference scheme for a porous-fractured-vuggy medium. The discrete nonlinear equations for water, oil, and gas flow at Node \( i \) are written as follows:

\[
\left\{ \left( M_{\beta}\right)_{ij} n + 1 - \left( M_{\beta}\right)_{ij} n \right\} \frac{V_i}{\Delta t} = \sum_{j \in \eta_i} F_{\beta,ij}^{n+1} + Q_{\beta i}^{n+1}
\]

where \( M \) is the mass accumulation term of phase \( \beta \); superscript \( n \) denotes the previous time level; \( n+1 \) is the current time level; \( V_i \) is the volume of element \( i \) (porous or fractured block); \( \Delta t \) is time step size; \( \eta_i \) contains the set of neighboring elements \( (j) \) (porous, vuggy, or fractured) to which element \( i \) is directly connected; \( F_{\beta,ij} \) is the mass flow term for phase \( \beta \) between elements \( i \) and \( j \); and \( Q_{\beta i} \) is the mass sink/source term at element \( i \), of phase \( \beta \).

The “flow” term \( F_{\beta,ij} \) in Equation (3) for multiphase flow between and among the triple-continuum media, along the connection \( (i,j) \), is given by

\[
F_{\beta,ij} = \lambda_{\beta,ij+1/2} \gamma_{ij} \left[ \psi_{\beta i} - \psi_{\beta j} \right]
\]

where \( \lambda_{\beta,ij+1/2} \) is the mobility term to phase \( \beta \), defined as

\[
\lambda_{\beta,ij+1/2} = \frac{\mu_{\beta ij+1/2}}{k_{ij+1/2}}
\]

Here subscript \( ij+1/2 \) denotes a proper averaging or weighting of properties at the interface between two elements \( i \) and \( j \); and \( \psi_{\beta i} \) is the relative permeability to phase \( \beta \). In Equation (7), \( \gamma_{ij} \) is transmissivity and is defined, when the integral finite-difference scheme is used, as

\[
\gamma_{ij} = \frac{A_{ij} k_{ij+1/2}}{D_i + d_j}
\]

where \( A_{ij} \) is the common interface area between connected blocks or nodes \( i \) and \( j \); \( d_i \) is the distance from the center of block \( i \) to the interface between blocks \( i \) and \( j \); and \( k_{ij+1/2} \) is an averaged (e.g., harmonically weighted) absolute permeability along the connection between elements \( i \) and \( j \). The flow potential term in Equation (4) is defined as

\[
\psi_{\beta i} = \rho_{\beta i} - \rho_{\beta,ij+1/2} g D_i
\]

where \( D_i \) is the depth to the center of block \( i \) from a reference datum. The mass sink/source term at element \( i \), \( Q_{\beta i} \), is defined as

\[
Q_{\beta i} = q_{\beta i} V_i
\]

Note that Equation (6) has the same form regardless of the dimensionality of the model domain, i.e., it applies to one-, two-, or three-dimensional analyses of multiphase flow through vuggy fractured porous media. In our numerical model, Equation (6) is written in a residual form and is fully implicitly solved using a Newton/Raphson iteration.

**Handling Fractures and Vugs**

The technique used in this work for handling multiphase flow through vuggy fractured rock follows the dual- or multi-continuum methodology. With this dual-continuum concept, Equations (1), (2), (3), and (4) can be used to describe multiphase flow along fractures and inside matrix blocks, as well as fracture-matrix-vug interaction. However, special attention needs to be paid to treating interporosity flow in the fracture-matrix-vug triple continua. Flow terms between fracture-matrix, fracture-vug, and vug-matrix connections are all evaluated using Equation (6). However, the transmissivity of (9) will be evaluated differently for different types of interporosity flow. For fracture-matrix flow, \( \gamma_{ij} \), is given by

\[
\gamma_{FM} = \frac{A_{FM} k_M}{l_{FM}}
\]

where \( A_{FM} \) is the total interfacial area between fractures (F) and the matrix (M) elements; \( k_M \) is the matrix absolute permeability; and \( l_{FM} \) is the characteristic distance for flow crossing fracture-matrix interfaces. For fracture-vug flow, \( \gamma_{ij} \) is defined as

\[
\gamma_{FV} = \frac{A_{FV} k_V}{l_{FV}}
\]

where \( A_{FV} \) is the total interfacial area between the fracture (F) and vugs (V) elements; \( l_{VM} \) is a characteristic distance for flow across vug-matrix interfaces; and \( k_V \) is the absolute vuggy permeability, which is the actual permeability of small
fractures that control flow between vugs and fractures (Figure 1). Note that for the case in which vugs are isolated from fractures, as shown in Figures 2 and 3, no fracture-vug flow terms need to be calculated, because they are indirectly connected through the matrix. For vug-matrix flow, $\gamma_{ij}$ is evaluated as

$$
\gamma_{VM} = \frac{A_{VM} k_m}{l_{VM}}
$$

(14)

where $A_{VM}$ is the total interfacial area between the vug (V) and matrix (M) elements; and $l_{VM}$ is a characteristic distance for flow crossing vug-matrix interfaces.

Note that Table 1 summarizes several simple models for estimating characteristic distances in calculating inter-porosity flow within fractures, vugs, and the matrix. In such cases, we have regular one-, two-, or three-dimensional large fracture networks, each with uniformly distributed small fractures connecting vugs or isolating vugs from fractures (Figures 1, 2, and 3). The models in Table 1 rely on the quasi-steady-state flow assumption of Warren and Root\textsuperscript{21} to derive characteristic distances for flow between fracture-matrix and (through small fractures) fracture-vug connections. Another condition for using the formulation in Table 1 is that fractures, vugs, and the matrix are all represented by only one grid block. In addition, the flow distance between large fractures (F) and vugs (V), when connected through small fractures, is taken to be half the characteristic length of the small fractures within a matrix block (Figure 1). Furthermore, the interface areas between vugs and the matrix should include the contribution of small fractures for the case of Figure 1. Interface areas between fractures and the matrix, and between fractures and vugs through connecting small fractures, should be treated using the geometry of the large fractures alone. This treatment implicitly defines the permeabilities of the fractures in a continuum sense, such that bulk connection areas are needed to calculate Darcy flow between the two fracture continua.

The MINC concept\textsuperscript{18} is extended to generate a triple-continuum grid, which is a key step in modeling flow through fractured-vuggy rock. We start with a primary or single-porous medium mesh that uses bulk volume of formation and layering data. Then, we use geometric information for the corresponding fractures and vugs within each formation subdomain or each finite-difference grid block of the primary mesh. Fractures are lumped together into the fracture continuum, while vugs with or without small fractures are lumped together into the vuggy continuum. The rest is treated as the matrix continuum. Connection distances and interface areas are then calculated accordingly, e.g., using the relations listed in Table 1 and the geometric data of fractures. Once a proper mesh for a triple-continuum system is generated, fracture, vuggy, and matrix blocks are specified, separately, to represent fracture or matrix continua.

Examination with Analytical Solution

We examined the numerical model using an analytical solution\textsuperscript{14, 22}. The verification problem concerns typical transient flow towards a well that fully penetrates a radially infinite, horizontal, and uniformly vuggy fractured reservoir. Numerically, a radial reservoir ($r_e = 10,000$ m) of $20$ m thick is represented by a 1-D (primary) grid of $2,100$ intervals. A triple-continuum mesh is then generated using a 1-D vuggy-fracture-matrix conceptual model, consisting of a horizontal large-fracture plate network with a uniform disk-shaped matrix block. Uniform spherical vugs are contained inside the matrix and connected to fractures through smaller fractures. Fracture, vugs and matrix parameters are given in Table 2.

Figure 4 compares numerical modeling results with the analytical solution for a single-phase transient flow case (in terms of dimensionless variables). Excellent agreement exists between the two solutions, which provides verification of the numerical formation and its implementation. Note that there are very small differences at very early times ($t_0 < 10$ or 0.2 seconds) in the two solutions of Figure 4, which may occur because the analytical solution, which is long-time asymptotic and similar to the Warren-Root solution, may not be valid for $t_0 < 100$.

Application to a Field Problem

The numerical model is here demonstrated to simulate oil and water production in a selected fracture-vug unit of Region 4 of Tahe Reservoir in western China. Geologic formation in Region 4 is typical of fractured, vuggy rock and oil reserves are mainly within pore space of fractures and varying-sized cavities or vugs. The reservoir conditions and parameters are: formation temperature is at 121°C; initial pressure = 595.8 bars; crude oil viscosity is 3.4 cp; STC oil density is 0.88 g/cm$^3$; original oil formation volume factor is 1.1; oil compressibility is $8.1 \times 10^{-9}$/Pa; water compressibility is $1.0 \times 10^{-10}$/Pa; and water density is 1.0 g/cm$^3$. Oil production started in 1997. Since then, a large amount of oil has been produced from Region 4.

A three-dimensional (3-D) numerical grid is generated to simulate this production unit and Figure 5 shows a plan view of the 3-D grid and the horizontal domain of the unit formation. Horizontally, the grid uses uniform grid blocks, with 45 columns in the x direction and 60 rows in the y direction. Vertically, the reservoir domain is subdivided into 16 geological layers (See Table 3) and discretized into 30 numerical grid layers with varying thickness.

Table 3 lists the geological and grid layers with rock properties used for the field simulation example. The rock properties were estimated from core, well-log, and other geophysical data. As shown in Table 3, the formation consists of four rock types: (1) ROCK1 for unfilled vug rock, (2) ROCK2 for partially filled vug rock; (3) ROCK3 for fully filled vug rock; and (4) DENSE for tight and low-permeability layer, which behaves as an aquitard, allowing only vertical flow between oil-production layers.

Figure 6 shows an example of the simulated current oil saturation distributions within the unit, after history matching. It is found that model results are reasonably matching measured water saturation data as well as water-cut data, which cannot be obtained without including vug effects.

Summary and Concluding Remarks

We have developed a physically based conceptual and numerical model for simulating multiphase flow through...
vuggy fractured rock using a triple-continuum medium approach. The proposed multicontinuum concept is a natural extension of the classic double-porosity model, with the fracture continuum responsible for conducting global flow, while vuggy and matrix continua, locally connected as well as interacting with globally connecting fractures, provide storage space.

The proposed conceptual model has been implemented into a general multidimensional numerical reservoir simulator using a control-volume, finite-difference approach, which can be used to simulate single-phase as well as multiple-phase flow in 1-D, 2-D and 3-D reservoirs. A verification example is provided for the numerical scheme by comparing numerical results against an analytical solution for single-phase flow. Furthermore, the model is demonstrated to modeling a fractured-vuggy reservoir and more simulation studies of actual vuggy-fractured petroleum reservoirs are under way.

Acknowledgments

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References


Table 1. Characteristic distances* for evaluating flow terms between fractures, vugs, and matrix systems

<table>
<thead>
<tr>
<th>Fracture Sets</th>
<th>Dimensions of Matrix Blocks (m)</th>
<th>Characteristic F-M Distances (m)</th>
<th>Characteristic F-V Distances (m)</th>
<th>Characteristic V-M Distances 1 (m)</th>
<th>Characteristic V-M Distances 2 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D A</td>
<td></td>
<td>$l_{FM} = A / 6$</td>
<td>$l_{FV} = l_x$</td>
<td>$l_{VM} = a / 6$</td>
<td>$l_{VM} = (A - d_x) / 2$</td>
</tr>
<tr>
<td>2-D A, B</td>
<td></td>
<td>$l_{FM} = AB / 4(A + B)$</td>
<td>$l_{FV} = l_x + l_y / 2$</td>
<td>$l_{VM} = ab / 4(a + b)$</td>
<td>$l_{VM} = A + B - 2d_c / 4$</td>
</tr>
<tr>
<td>3-D A, B, C</td>
<td></td>
<td>$l_{FM} = 3ABC / 10(AB + BC + CA)$</td>
<td>$l_{FF} = l_x + l_y + l_z / 3$</td>
<td>$l_{VM} = 3abc / 10 / (ab + bc + ca)$</td>
<td>$l_{FF} = A + B + C - 3d_c / 6$</td>
</tr>
</tbody>
</table>

*Note in Table 1, A, B, and C are dimensions of matrix blocks along x, y, and z directions, respectively.

Characteristic V-M distances are estimated for the case (Figure 1), i.e., vuggy-matrix connections are dominated by small fractures, where dimensions a, b, and c are fracture-spacings of small fractures along x, y, and z directions, respectively.

Characteristic V-M distances are used for the case (Figures 2 and 3), i.e., vugs are isolated from fractures.

Table 2. Parameters used in the single-phase flow problem of the triple-continuum, vuggy fractured reservoir

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix porosity</td>
<td>$\phi_M = 0.263$</td>
<td></td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>$\phi_F = 0.001$</td>
<td></td>
</tr>
<tr>
<td>Vuggy porosity</td>
<td>$\phi_V = 0.01$</td>
<td></td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>A = 5</td>
<td>M</td>
</tr>
<tr>
<td>Small-fracture spacing</td>
<td>a = 1.6</td>
<td>M</td>
</tr>
<tr>
<td>F characteristic length</td>
<td>$l_F = 3.472$</td>
<td>M</td>
</tr>
<tr>
<td>F-M/F-V areas per unit volume rock</td>
<td>$A_{FM} = A_{FV} = 0.61$</td>
<td>m$^2$/m$^3$</td>
</tr>
<tr>
<td>Reference water density</td>
<td>$\rho_i = 1,000$</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Water phase viscosity</td>
<td>$\mu = 1 \times 10^{-3}$</td>
<td>Pa•s</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>$k_M = 1.572 \times 10^{-16}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>$k_F = 1.383 \times 10^{-13}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Small-fracture or vug permeability</td>
<td>$k_V = 1.383 \times 10^{-14}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Water Production Rate</td>
<td>$q = 100$</td>
<td>m$^3$/d</td>
</tr>
<tr>
<td>Total compressibility of three media</td>
<td>$C = C_M = C_F = 1.0 \times 10^{-10}$</td>
<td>1/Pa</td>
</tr>
<tr>
<td>Well radius</td>
<td>$r_w = 0.1$</td>
<td>m</td>
</tr>
<tr>
<td>Formation thickness</td>
<td>h = 20</td>
<td>m</td>
</tr>
</tbody>
</table>
Table 3. Geological and grid layering and rock properties used for the field simulation example

<table>
<thead>
<tr>
<th>Layer</th>
<th>Top Depth (ft)</th>
<th>Bottom depth (ft)</th>
<th>Thickness (ft)</th>
<th>Fracture Porosity (%)</th>
<th>Vug-cavity Porosity (%)</th>
<th>Oil saturation</th>
<th>Rock type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5346.1</td>
<td>5347.4</td>
<td>1.3</td>
<td>0.08</td>
<td>26.55</td>
<td>88.48</td>
<td>ROCK1</td>
</tr>
<tr>
<td>2</td>
<td>5347.4</td>
<td>5356.5</td>
<td>9.1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>DENCE</td>
</tr>
<tr>
<td>3</td>
<td>5356.5</td>
<td>5372.4</td>
<td>15.9</td>
<td>0.35</td>
<td>4.57</td>
<td>71.33</td>
<td>ROCK2</td>
</tr>
<tr>
<td>4</td>
<td>5372.4</td>
<td>5401.3</td>
<td>28.9</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>DENCE</td>
</tr>
<tr>
<td>5</td>
<td>5401.3</td>
<td>5414.8</td>
<td>13.5</td>
<td>0.15</td>
<td>1.21</td>
<td>60.00</td>
<td>ROCK3</td>
</tr>
<tr>
<td>6</td>
<td>5414.8</td>
<td>5429.8</td>
<td>15</td>
<td>0.16</td>
<td>1.01</td>
<td>72.27</td>
<td>ROCK3</td>
</tr>
<tr>
<td>7</td>
<td>5429.8</td>
<td>5430.9</td>
<td>1.1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>DENCE</td>
</tr>
<tr>
<td>8</td>
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Figure 1. Conceptualization#1 of vuggy fractured rock as a triple-continuum system with vugs indirectly connected to fractures through small fractures.
Figure 2. Conceptualization#2 of vuggy fractured rock as a triple-continuum system with vugs isolated from or indirectly connected to fractures through rock matrix.

Figure 3. Conceptualization#3 of vuggy fractured rock as a triple-continuum system with partial vugs isolated from and certain vugs directly connected to fractures.
Figure 4. Comparison between analytical and numerical solutions for single-phase transient flow through vuggy fractured formation.

Figure 5. Plan view of the 3-D numerical grid used in the field simulation example.
Figure 6. Oil saturation and its distributions simulated within the fractured-vug unit of Region 4 of Tahe Reservoir