A novel fully-coupled flow and geomechanics model in enhanced geothermal reservoirs

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**A B S T R A C T**

The geomechanical behavior of porous media has become increasingly important in stress-sensitive reservoirs. This paper presents a novel fully-coupled fluid-flow-geomechanical model (TOUGH2-EGS). The fluid flow portion of our model is based on the general-purpose numerical simulator TOUGH2-EOS3. The geomechanical portion is developed from linear elastic theory for a thermo-poro-elastic system using the Navier equation. Fluid flow and geomechanics are fully coupled, and the integral finite-difference method is used to solve the flow and stress equations. In addition, porosity and permeability depend on effective stress and correlations describing that dependence are incorporated into the simulator. TOUGH2-EGS is verified against analytical solutions for temperature-induced deformation and pressure-induced flow and deformation. Finally the model is applied to analyze pressure and temperature changes and deformation at The Geysers geothermal field. The results demonstrate that the model can be used for field-scale reservoir simulation with fluid flow and geomechanical effects.

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1. Introduction

In the past, reservoir engineers have neglected geomechanical effects when considering porous media fluid flow because of little change in rock properties or deformation in conventional reservoirs. However, geomechanical effects should not be ignored in many instances related to enhanced geothermal systems, such as analyzing high flow rate drive oil recovery, associated formation subsidence, stress sensitive fractured reservoirs, and dealing with wellbore stability, and production of heavy oil (Merle et al., 1976; Kosloff et al., 1980; Lewis and Schrefler, 1998; Settari and Walters, 2001; Samier and Gennaro, 2008; Boull et al., 2011). The coupling of geomechanics with porous media fluid and heat flow is of importance in a variety of technical venues. Some examples are soil shrinkage from water evaporation and soil heaving due to water freezing; formation permeability and porosity changes and ground surface uplift from subsurface CO\textsubscript{2} sequestration in a saturated geologic formation; and rock deformation associated with heavy oil recovery processes such as steam assisted gravity drainage or from cold water injection and hot water production in geothermal fields.

Models with coupled flow and geomechanics can be classified into four types: fully coupled, iteratively coupled, explicitly coupled, and loosely coupled (Settari and Walters, 2001; Longuemare et al., 2002; Minkoff et al., 2003; Tran et al., 2004; Samier and Gennaro, 2008). For a fully coupled simulator, a set of equations (generally a large system of nonlinear coupled partial differential equations) incorporating all of the relevant physics must be derived (Minkoff et al., 2003). The iterative coupling method solves the pore fluid flow variables and the geomechanical conditions independently and sequentially. The iterative coupling between the reservoir simulator and the geomechanical model is then performed at the end of each timestep through pore volume calculations (Longuemare et al., 2002). The explicitly coupled method is a special case of the iteratively coupled method in which only one iteration per one time increment is performed. In loose coupling, there are two sets of equations which are solved independently, but information is passed at designated time intervals in both directions between fluid flow and geomechanics simulators (Minkoff et al., 2003). In the fully coupled method the equations of flow and geomechanics are solved simultaneously at each time step. In the iteratively coupled method, either the flow or mechanical problem is solved first, and then the other equation is solved iteratively using that intermediate solution information (GEOSIM (Settari et al., 2000), GeoSyS/Rockflow (Wang and Kolditz, 2007)). In explicitly coupled methods, only one iteration is taken between the geomechanical and fluid flow modules, for example, ROCMAS (Noorishad et al., 1984).
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{nm}$</td>
<td>the cross area, m²</td>
</tr>
<tr>
<td>$A_j$</td>
<td>the cross area of grid $j$, m²</td>
</tr>
<tr>
<td>$A_{ij}$</td>
<td>the cross area between grid $i$ and $j$, m²</td>
</tr>
<tr>
<td>$C_k$</td>
<td>heat conductivity, W K⁻¹ m⁻¹</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat capacity of rock, J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Bulk total compressibility, Pa⁻¹</td>
</tr>
<tr>
<td>$D_T$</td>
<td>Thermal diffusivity, m² s⁻¹</td>
</tr>
<tr>
<td>$E$</td>
<td>Young modulus, Pa</td>
</tr>
<tr>
<td>$F$</td>
<td>the force, Pa</td>
</tr>
<tr>
<td>$F_m$</td>
<td>the mass or energy transport terms along the borehole due to advective processes, W m⁻¹</td>
</tr>
<tr>
<td>$F_{nm}$</td>
<td>the mass or energy transport terms along cross section nm due to advective processes, W m⁻¹</td>
</tr>
<tr>
<td>$F_l$</td>
<td>l-direction body force (gravity), Pa m⁻¹</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration constant, m s⁻²</td>
</tr>
<tr>
<td>$h$</td>
<td>the total column height, m</td>
</tr>
<tr>
<td>$h_β$</td>
<td>Specific enthalpy in phase $β$, J kg⁻¹</td>
</tr>
<tr>
<td>$k$</td>
<td>Absolute permeability, m²</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Heat conductivity of rock W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>$K$</td>
<td>Bulk modulus, Pa</td>
</tr>
<tr>
<td>$k_{β,β}$</td>
<td>Relative permeability to phase $β$</td>
</tr>
<tr>
<td>$M$</td>
<td>Biot's modulus, Pa</td>
</tr>
<tr>
<td>$M^*$</td>
<td>the accumulation terms of the components and energy $κ$, kg m⁻³</td>
</tr>
<tr>
<td>$M^*_n$</td>
<td>the accumulation terms of the components and energy $κ$ of grid $n$, kg m⁻³</td>
</tr>
<tr>
<td>$n$</td>
<td>Normal vector on surface element, dimensionless</td>
</tr>
<tr>
<td>$T$</td>
<td>Time, s</td>
</tr>
<tr>
<td>$T_{ref}$</td>
<td>Reference temperature, °C or K</td>
</tr>
<tr>
<td>$u_β$</td>
<td>the Darcy velocity in phase $β$, m s⁻¹</td>
</tr>
<tr>
<td>$U_β$</td>
<td>the internal energy of phase $β$ per unit mass, J kg⁻¹</td>
</tr>
<tr>
<td>$V_n$</td>
<td>Volume of the nth grid cell, m³</td>
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<tr>
<td>$P$</td>
<td>Pressure, Pa</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Incremental pressure due to load, Pa</td>
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<td>$P_c$</td>
<td>Capillary pressure, Pa</td>
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<tr>
<td>$P_{ref}$</td>
<td>Reference capillary pressure, Pa</td>
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<tr>
<td>$p$</td>
<td>the fluid pressure in phase $β$, Pa</td>
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<tr>
<td>$q_m$</td>
<td>Source/sink terms for mass or energy components, kg m⁻³ s⁻¹</td>
</tr>
<tr>
<td>$q_m^*$</td>
<td>Source/sink terms for mass or energy components of grid $n$, kg m⁻³ s⁻¹</td>
</tr>
<tr>
<td>$R_n^*$</td>
<td>the residual of component for grid block $n$, kg s⁻¹</td>
</tr>
<tr>
<td>$R_n$</td>
<td>the residual of stress for grid block $n$, Pa m⁻²</td>
</tr>
<tr>
<td>$S$</td>
<td>Storage coefficient, Pa⁻¹</td>
</tr>
<tr>
<td>$S_l$</td>
<td>Saturation of liquid phase, dimensionless</td>
</tr>
<tr>
<td>$S_p^*$</td>
<td>Saturation of phase $β$, dimensionless</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Constant temperature boundary, °C</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Initial temperature, °C</td>
</tr>
<tr>
<td>$w$</td>
<td>Vertical displacement of the upper surface, m</td>
</tr>
<tr>
<td>$x^i$</td>
<td>Primary variables at time $t$, pressure, temperature, air fraction, or stress</td>
</tr>
<tr>
<td>$X_β^*$</td>
<td>Mass fraction of component $β$ in fluid phase $β$, dimensionless</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Bulk volume, m³</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance along-column coordinate, m</td>
</tr>
</tbody>
</table>

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$α$</td>
<td>Biot’s coefficient, dimensionless</td>
</tr>
<tr>
<td>$α_p$</td>
<td>Biot’s coefficient, dimensionless</td>
</tr>
<tr>
<td>$α_f$</td>
<td>Biot’s coefficient, dimensionless</td>
</tr>
<tr>
<td>$β$</td>
<td>Linear thermal expansion coefficient, °C⁻¹</td>
</tr>
<tr>
<td>$μ$</td>
<td>Viscosity, Pa s</td>
</tr>
<tr>
<td>$μ_f$</td>
<td>fluid viscosity, Pa s</td>
</tr>
<tr>
<td>$ϕ$</td>
<td>Porosity, dimensionless</td>
</tr>
<tr>
<td>$λ$</td>
<td>Thermal conductivity, W K⁻¹ m⁻¹</td>
</tr>
<tr>
<td>$λ_s$</td>
<td>Lame’s constant, Pa</td>
</tr>
<tr>
<td>$ε_β$</td>
<td>Strain components, $l=x, y, z$, dimensionless</td>
</tr>
<tr>
<td>$ε_{β,l}$</td>
<td>Strain components, $l=xy, yz, zx$, dimensionless</td>
</tr>
<tr>
<td>$ε_{β,j}$</td>
<td>Strain components, $j=x, y, z$, dimensionless</td>
</tr>
<tr>
<td>$ε_{v,l}$</td>
<td>Volumetric strain, dimensionless</td>
</tr>
<tr>
<td>$Π$</td>
<td>Strain tensor, dimensionless</td>
</tr>
<tr>
<td>$u_l$</td>
<td>Displacement component, $l=x,y,z$, m</td>
</tr>
<tr>
<td>$ρ$</td>
<td>the index for the components, $κ=1$ (water), $2$ (air), and $3$ (energy)</td>
</tr>
<tr>
<td>$β$</td>
<td>G for gas; L for liquid</td>
</tr>
</tbody>
</table>

The fully coupled method is internal consistent and rigorous, because the fluid flow and geomechanical equations are solved simultaneously on the same discretized grid. Consequently, considerable effort is required to develop such a simulator (Settari and Walters, 2001). Typical geomechanical models assign rock displacement or velocity as primary variables, two primary variables for 2-D and three primary variables for 3-D. As a result, the coupling flow-geomechanical model requires intensive computation. Various coupling techniques have been developed to reduce such computational time required.

The objective of this paper is to present a new fully-coupled multiphase, heat flow and geomechanical model, including the mathematical equations and formulation. Mean total stress is the
only primary variable for geomechanical model in a 3-D problem. Thus, the computational requirement is less than that of typical geomechanical model. We then verify the model using two analytical solutions, and finally apply the model to a field example. It is assumed that the reservoir rock is linear elastic and obeys the generalized version of Hooke’s law. The novelty of our model lies in its simplified treatment of rock deformation using the relation of mean normal stress and volumetric strain. Pressure, temperature, air mass fraction, and mean total stress are solved simultaneously for each Newton iteration. The advantages of the simplification of typical geomechanical model lies on (1) the computational requirement is less than that of the typical geomechanical model because of less primary variable and (2) this method is still capable of capturing geomechanical behavior of rock as seen in the comparison between numerical and analytical solution as well as in Geyser case.

2. Mathematical and numerical model

2.1. Multiphase fluid and heat flow module

The basis for our simulator is the TOUGH2/EOS3 model, which solves the mass and energy balance equations describing fluid and heat flow in multiphase, multi-component systems. Fluid flow is governed by a multiphase extension of Darcy’s law and there is diffusive mass transport in all phases. Heat flow occurs by conduction and convection, with sensible as well as latent heat effects included. The TOUGH2/EOS3 mass and energy equations for two-phase flow of two components (water, air) are summarized in Table 1 (see Nomenclature for definitions of all symbols used).

2.2. Geomechanical module

This fully coupling assumes that boundaries of each element can move as an elastic material and obey the generalized version of Hooke’s law (Winterfeld and Wu, 2011). The mean total stress is an additional primary variable. Under the assumption of linear elasticity with small strains for a thermoporoelastic system, the equilibrium equation can be expressed as follows: (Jaeger et al., 2007)

$$\tau_{ii} = \left(\alpha P + 3\beta K(T - T_{ref})\right) = 2\nu F_{yy} + \nu (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \quad \text{for } i, j = x, y, z$$

(1)

and the shears are

$$\tau_{ij} = 2\nu \epsilon_{ij} \quad \text{for } i, j = x, y, z \quad \text{and } i \neq j$$

(2)

The trace of Hooke’s law for a thermoporoelastic medium is

$$K_{xy} = \epsilon_{xy} - \alpha P - 3\beta K(T - T_{ref})$$

(3)

These stress and strains are symmetric tensors. The equations of stress equilibrium are derived from a force balance on a differential volume element. They are, when neglecting acceleration

$$\frac{\partial \tau_{ii}}{\partial x} + \frac{\partial \tau_{ij}}{\partial y} + \frac{\partial \tau_{ij}}{\partial z} + F_i = 0, \quad i, j = x, y, z$$

(4)

Strain can be expressed in terms of a displacement vector, u. The displacement vector points from the new position of a volume element to its previous one. The strain tensor is related to the displacement vector by

$$\epsilon = \frac{1}{2} \left[ \nabla u + (\nabla u)^T \right]$$

(5)

Each element of Eq. (5) has the form

$$\epsilon_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right], \quad (i, j) = (x, y, z); \quad x_i = x, y, z$$

(6)

We then derive (see Appendix A)

$$3(1-\nu) \nu^2 \tau_{xx} + \nu \cdot F - 2(1-2\nu) (\nu^2 P + 3\beta K \nu^2 T) = 0$$

(7)

The boundary type of stress includes specified stress boundary only. Specified stress boundary remains fixed at all time steps and the mean total stress at other places will be subjected to the pressure, temperature and body force. For 1D, 2D and 3D cases, the model should include at least 1, 2 and 3 specified stress boundaries respectively. For an example of 1D case, the model is discretized into N gridblocks, and the number of connections should be N−1. From Eq. (7), there are N−1 equations, and the number of boundary condition should be at least 1, and so we get N equations and N unknowns of mean total stress. So we can obtain mean normal stress field after solving the linear equations.

2.3. Space and times discretization

The integral finite-difference method (Pruess et al., 1999) is used to discretize the continuous space variables for numerical simulation. Time discretization is carried out using a backward, first-order, and fully implicit finite-difference scheme. Time and space discretization of the governing mass and energy balance equations results in a set of coupled non-linear equations, which can be written in residual form as follows:

$$R_{\phi}(x_{n+1}) = M_{\phi}(x_{n+1}) - M_{\phi}(x_{n}) \frac{\Delta t}{\Delta x} \left( \sum_{n} A_{nm} \frac{\phi_{n+1}}{\Delta x} + V_{\phi_{n+1}} \right) = 0, \quad \phi = 1, 2, 3 \quad \text{in each block}$$

(8)

The stress equation, Eq. (7), relates mean total stress to pore pressure, temperature, and body forces. It is also discretized using the integral finite difference method over a volume element with an outer surface. After applying the divergence theorem to the integral finite difference volume integral the following is obtained:

$$\int \left( \frac{3}{1+\nu} \nu \tau_{xx} + \nu \cdot F - 2(1-2\nu) (\nu^2 P + 3\beta K \nu^2 T) \right) \hat{n} d\Gamma = 0$$

(9)

The surface integral is expressed as a discrete sum of averages over surface segments

$$\sum_{j} \left( \frac{3}{1+\nu} \nu \tau_{xx} + \nu \cdot F - 2(1-2\nu) (\nu^2 P + 3\beta K \nu^2 T) \right) A_j = 0$$

(10)

The boundary conditions for Eq. (10) are a reference temperature, pressure, and stress at some distance from a given grid block. For example, surface pressure, stress (both atmospheric) and temperature can be used as boundary conditions.

The finite difference approximation for Eq. (10) in residual form is

$$R_{\phi}(x_{n+1}) = \sum_{j} \left( \frac{3}{1+\nu} \nu \tau_{xx} + \nu \cdot F - 2(1-2\nu) (\nu^2 P + 3\beta K \nu^2 T) \right) A_j = 0$$

(11)

The model solves for four primary variables (pressure, air mass fraction/gas saturation, temperature, and mean total stress) for each
grid block. A uniform residual form for four primary variables is shown in Eq. (12). For variables of pressure, air mass fraction/gas saturation and temperature, the residuals are formed from Eq. (8). For mean total stress, the residuals is assembled from Eq. (11).

Eq. (12) are solved by the Newton/Raphson method that iterates until the residuals are reduced below preset convergence criteria.

\[-\sum_{i} \frac{\partial R_{n,i}^{\tau+1}}{\partial x_{i}} (x_{p,\tau+1} - x_{i,\tau}) = R_{n,\tau+1}(x_{p,\tau}). \quad \kappa = 1, 2, 3, 4\]  

(12)

2.4. Stress-dependent parameters modification

Permeability and porosity are both dependent on effective stress according to the following relations:

\[\alpha' = \tau_n - aP\]  

(13)

\[k = k(\alpha', p)\]  

(14)

\[\phi = \phi(\alpha', p)\]  

(15)

Since bulk volume is related to porosity, bulk volume depends on effective stress and pore pressure

\[V_b = V_s(\alpha', P)\]  

(16)

In addition, permeability and porosity are used to scale capillary pressure according to the relation by Leverett (1941)

\[P_c = P_c(\sqrt{k/\phi})\]  

(17)

where subscript 0 refers to reference conditions.


2.5. Model architecture of TOUGH2-EGS

The model architecture is summarized in Fig. 1. There are four primary variables, pressure, temperature, air mass fraction, and mean total stress. Secondary variables such as liquid saturation and volumetric strain are derived from the primary variables. First, the grid systems, boundary conditions, sources and sink terms, initialization parameters of pressure, temperature, and mean total stress are inputted to the model. The initial stress field is then calculated based on Eq. (10) with initial pressure field, initial temperature field and stress boundary conditions. Then, the time iteration is carried out. During time iteration, the coefficient matrices for four primary variables (\(\kappa = 1, 2, 3, 4\)) in Eq. (12) are assembled, and then pressure, temperature, air mass fraction, and mean total stress are solved iteratively for each Newton iteration. Also, permeability and porosity correction will be carried out in each time iteration as the module of equation of state. The calculation of fluid and geomechanical variables is fully implicit and fully coupled.

3. Verification of TOUGH2-EGS

Consolidation problems subjected to stress and temperature change will be verified. Here, three cases, 1-D consolidation, 1-D heat conduction and 2-D consolidation (Mandel’s problem), is selected for testing the applicability of our model when comparing the simulated results with analytical solutions. The poroelastic was verified by comparing the numerical result against the analytical solution of 1-D consolidation problem and the thermoelastic was verified against the analytical solution of 1-D heat conduction.

3.1. 1-D consolidation in a porous permeable column

A 1-D consolidation problem is simulated numerically and compared with Jaeger’s analytical solution (Jaeger et al., 2007, listed in Appendix D) to verify the validity of the simulator code. The 1-D problem is a porous permeable column that undergoes uniaxial strain in the vertical direction only. The column is subjected to a constant load on the top (Fig. 2), the fluid boundary pressure is set to zero gauge right after the load is imposed, and only vertical displacement takes place. Basic parameters for rock properties, fluid properties, initial and boundary conditions can be seen in Fig. 2 and are listed in Table 2.

The comparison of normalized excess pressure (defined as the ratio of pressure change to the maximum pressure) and vertical displacement between the analytical and numerical solutions in Fig. 3 indicates that our numerical results produce essentially the same answers as analytical models, which lend credibility to our computational approach.

3.2. 1-D heat conduction in a deformable rock column

1-D heat conduction in a deformable medium is simulated numerically and compared with Jaeger’s analytical solution (Jaeger et al., 2007, listed in Appendix E) to verify the validity of the simulator code. The 1-D problem is a non-permeable column that undergoes uniaxial strain in the vertical direction only. The column is subjected to a constant temperature on the top (Fig. 4) and only heat conduction takes place. Input parameters
Fig. 2. Problem description of 1-D consolidation.

Table 2
Input parameters used in simulation of the 1-D consolidation problem.

<table>
<thead>
<tr>
<th>Model</th>
<th>1000 x 1 x 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid size</td>
<td>Δx=1, Δy=0.5, Δz=0.5 m</td>
</tr>
<tr>
<td>Rock properties</td>
<td></td>
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<tr>
<td>Porosity</td>
<td>0.094</td>
</tr>
<tr>
<td>Permeability</td>
<td>10^-13 m²</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>0.0 Pa^-1</td>
</tr>
<tr>
<td>Rock mechanics properties</td>
<td></td>
</tr>
<tr>
<td>Biot's coefficient</td>
<td>1.0</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>5.0 x 10^9 Pa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Reference temperature</td>
<td>60 °C</td>
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<tr>
<td>Initial condition</td>
<td>2.466 x 10^6 Pa</td>
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<tr>
<td>Sink</td>
<td></td>
</tr>
<tr>
<td>Well index</td>
<td>2.0 x 10^-10 m³/(Pa.s)</td>
</tr>
<tr>
<td>Bottom hole pressure</td>
<td>1 x 10^5 Pa</td>
</tr>
</tbody>
</table>

Fig. 2. Problem description of 1-D consolidation.

Fig. 3. Comparisons between numerical and analytical solutions (a) pressure profiles and (b) displacement at the top of the column.

Table 3
Input parameters used in simulation of the 1-D heat conduction in a deformable rock column problem.

<table>
<thead>
<tr>
<th>Model</th>
<th>1 x 1 x 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid size</td>
<td>Δx=1, Δy=1, Δz=0.5 m</td>
</tr>
<tr>
<td>Rock properties</td>
<td></td>
</tr>
<tr>
<td>Porosity</td>
<td>0.01</td>
</tr>
<tr>
<td>Permeability</td>
<td>0.0 m²</td>
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<tr>
<td>Heat conductivity</td>
<td>2.34 W/m K</td>
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<tr>
<td>Heat capacity of rock</td>
<td>690 J/kg K</td>
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<tr>
<td>Density</td>
<td>2550 kg/m³</td>
</tr>
<tr>
<td>Rock mechanics properties</td>
<td></td>
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<tr>
<td>Linear thermal expansion</td>
<td>1.5 x 10^-6 K^-1</td>
</tr>
<tr>
<td>Bulk modulus</td>
<td>8.0 x 10^9 Pa</td>
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<tr>
<td>Poisson's ratio</td>
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<td>Initial condition</td>
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</tr>
<tr>
<td>Boundary condition</td>
<td></td>
</tr>
<tr>
<td>Constant temperature at the top</td>
<td>10 °C</td>
</tr>
</tbody>
</table>

Fig. 4. Problem description of 1-D heat conduction.

Fig. 5. Comparisons between numerical and analytical solutions (a) pressure profiles and (b) displacement at the top of the column.
in the model are listed in Table 3. Fig. 5a and b shows the match between analytical and numerical solutions are excellent.

3.3. Mandel’s problem

The geometry of Mandel’s problem is depicted in Fig. 6. An infinitely long specimen with a rectangular cross-section is sandwiched between two rigid, frictionless and impermeable plates. The specimen consists of incompressible solid constituents, and it is saturated with a single-phase incompressible fluid. At initial time, a force of 2F per unit thickness of the specimen is applied at the top and bottom. The lateral boundary surfaces perpendicular to the x direction are traction free and exposed to the ambient pressure. As predicted by the Skempton effect, a uniform pressure rise will be observed inside the specimen upon the exertion of a force 2F on the rigid plates. As time passes on, pore pressure near these boundaries must dissipate due to drainage access. Abousleiman et al. (1996) extend the classical problem to account for transversely isotropic material (the analytical solution is shown in Appendix F). Table 4 gives the dimensions of the specimen and its material properties used in this simulation (Fakcharoenphol et al., 2012). Fig. 7a and b shows the comparison of pressure at the center of the specimen, volumetric strain at the right and top edge of the specimen between numerical and analytical solutions. The pressure curve at the center has a peak and shows a good agreement with analytical solutions.

![Fig. 6. Mandel’s problem description.](image)

Fig. 6. Mandel’s problem description.

### Table 4

<table>
<thead>
<tr>
<th>Input parameters for Mandel’s problem.</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Model</td>
<td>1000 × 1000 × 100</td>
</tr>
<tr>
<td>Grid size</td>
<td>Δx = 10, Δy = 10, Δz = 100 m</td>
</tr>
<tr>
<td>Size</td>
<td>1000 × 1000 m²</td>
</tr>
<tr>
<td>Applied stress</td>
<td>1470 MPa</td>
</tr>
<tr>
<td>Rock and fluid properties</td>
<td></td>
</tr>
<tr>
<td>Porosity</td>
<td>0.094</td>
</tr>
<tr>
<td>Permeability</td>
<td>1.0e⁻¹³ m²</td>
</tr>
<tr>
<td>Pore compressibility</td>
<td>0.0</td>
</tr>
<tr>
<td>Rock mechanics properties</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>5.0 × 10¹⁰ Pa.s</td>
</tr>
<tr>
<td>Biot’s coefficient</td>
<td>1.0</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Initial condition</td>
<td></td>
</tr>
<tr>
<td>Initial pressure</td>
<td>3.0 × 10⁶ Pa</td>
</tr>
</tbody>
</table>

4. Model application

The Geysers is the site of the largest geothermal electricity generating operation in the world and has been in commercial production since 1960 (Mossop and Segall, 1997, 1999; Rutqvist and Tsang, 2002; Rutqvist et al., 2006a, 2006b; Rutqvist and Oldenburg, 2008; Khan and Truschel, 2010; Rutqvist, 2011). It is a vapor-dominated geothermal reservoir system that is hydraulically confined by low permeability rock. As a result of high steam withdrawal rates, the reservoir pressure declined until the mid 1990s, when increasing water injection rates resulted in a stabilization of the steam reservoir pressure. Archival INSAR images were acquired from approximately monthly satellite passes over the region for a seven-year period, seven-year period, from 1992 to 1999, and the data is compared with displacement calculated from our model.

The combined effects of steam production and water injection in 44 years and their influences on the ground deformation will be analyzed. Based on the work by Rutqvist et al. (2008) and Rutqvist et al. (2010), a cross-axis (NE-SW) two-dimensional model grid of the Geysers Geothermal Field was established. Permeability, temperature, and boundary conditions are shown in Fig. 8. The initial thermal and hydrological conditions (vertical distributions of temperature, pressure and liquid saturation) are typically established through steady-state multi-phase flow simulations. According to previous studies, the adopted rock-mass bulk modulus is 3 GPa and the linear thermal expansion coefficient is 3 × 10⁻⁵ °C⁻¹. Pore compressibility and the reservoir Poisson’s ratio of the reservoir is 1.0 × 10⁻¹⁰ Pa⁻¹ and 0.25, respectively. The injection well is about 217.5 m away from the production well. The steam-production and water-injection rate used in the model is estimated from the field-wide production and injection data (Mossop and Segall, 1997; Majer and Peterson, 2007; Khan and Truschel, 2010; Sanyal and Enedy, 2011).

4.1. Change of pressure and temperature after 44 years

Fig. 9 shows calculated liquid saturation and changes in fluid pressure and temperature after 44 years of production and injection. Fig. 9a shows the injection caused formation of a wet zone that extends towards 1000 m. Fig. 9b demonstrates pressure decrement is about 2 × 10⁶ Pa after steam production and water injection. Fig. 9c indicates a local cooling effect and the maximum

![Fig. 7. Comparison of pressure, volumetric strain between numerical and analytical solutions, (a) pressure, (b)volumetric strain.](image)
temperature decrement is about 50 °C. All the results are almost the same as the results from Rutqvist et al. (2008).

4.2. Changes in stress and volumetric strain

Fig. 10a and b displays changes in mean total stress and volumetric strain, respectively. The mean total stress change in the rock mass depends on the production-induced depletion and injection-induced cooling. The change in mean total stress is about 0.5–1.5 MPa and the volumetric strain is about 0.0001–0.0004. It is assumed that the length in x, y, and z directions will be changed uniformly and the ground displacement can be obtained from the volumetric strain and volume. Fig. 11 shows the change of simulated ground displacement with time and the comparison
paper is mainly concerned with fluid and heat flow and geomechanics in porous media, and geomechanics in the fractured reservoir is not discussed here.

Acknowledgments

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Appendix A. Derivations of geomechanical equation

Substituting Eqs. (1) and (6) into Eq. (5) and rearranging yields the following for \( x \)-component, \( y \)-component, and \( z \)-component, respectively:

\[
\alpha \frac{\partial P}{\partial x} + 3\beta K\frac{\partial T}{\partial x} + (G + \lambda_0) \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z} \right) \\
+ G \left( \frac{\partial^2 u_x}{\partial x \partial y} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial x \partial z} \right) + F_x = 0 \quad (A1)
\]

\[
\alpha \frac{\partial P}{\partial y} + 3\beta K\frac{\partial T}{\partial y} + (G + \lambda_0) \left( \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial y \partial z} \right) \\
+ G \left( \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial y \partial z} + \frac{\partial^2 u_y}{\partial y^2} \right) + F_y = 0 \quad (A2)
\]

\[
\alpha \frac{\partial P}{\partial z} + 3\beta K\frac{\partial T}{\partial z} + (G + \lambda_0) \left( \frac{\partial^2 u_z}{\partial z^2} + \frac{\partial^2 u_z}{\partial x \partial z} + \frac{\partial^2 u_z}{\partial y \partial z} \right) \\
+ G \left( \frac{\partial^2 u_z}{\partial x \partial z} + \frac{\partial^2 u_z}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2} \right) + F_z = 0 \quad (A3)
\]

Eqs. (A1)-(A3) can be expressed in vector notation as

\[
\alpha \nabla P + 3\beta K \alpha \nabla T + (\lambda_0 + G) \nabla (\nabla \cdot \mathbf{u}) + \nabla \cdot (\nabla P) + F = 0
\]

which is the thermoporoelastic version of the Navier equations.

Take the partial derivative with respect to \( x \) of \( x \)-component Eq. (A1), and the analogous for Eqs. (A2) and (A3), and add the three equations to obtain

\[
\alpha \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} \right) + \alpha \epsilon_x + \alpha \epsilon_y + \alpha \epsilon_z + 3\beta K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \\
+ (G + \lambda_0) \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y \partial x} + \frac{\partial^2 u_x}{\partial z \partial x} \right) + G \left( \frac{\partial^2 u_x}{\partial y \partial x} + \frac{\partial^2 u_x}{\partial z \partial x} + \frac{\partial^2 u_x}{\partial x \partial y} \right) \\
+ (G + \lambda_0) \left( \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial x \partial z} \right) + G \left( \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial z \partial y} + \frac{\partial^2 u_y}{\partial y \partial z} \right) \\
+ (G + \lambda_0) \left( \frac{\partial^2 u_z}{\partial x \partial z} + \frac{\partial^2 u_z}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2} \right) + G \left( \frac{\partial^2 u_z}{\partial x \partial z} + \frac{\partial^2 u_z}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2} \right) = 0
\]

Eq. (A5) written in vector notation is

\[
\alpha \nabla^2 P + 3\beta K \nabla^2 T + (\lambda_0 + 2G) \nabla (\nabla \cdot \mathbf{u}) + \nabla \cdot (\nabla P) + F = 0
\]

The divergence of the displacement vector is the volumetric strain

\[
\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \epsilon_v
\]

Summing Eq. (1) over \( x, y, \) and \( z \)-components gives the trace of Hooke’s law for a thermoporoelastic medium. This sum is an
invariant for an isotropic solid, and is
\[
\left(\lambda + \frac{2}{3}\mu\right)\tau_{ij} = \frac{\tau_{ik} + \tau_{kj} + \tau_{ij}}{3} - \alpha P - 3\beta K(T - T_0) + \tau_{im} - \alpha P - 3\beta K(T - T_0)
\]
(A8)

Substituting Eqs. (A7) and (A8) into Eq. (A6) yields
\[
\alpha\nabla^2 P + 3\beta K\nabla^2 T + \left(\lambda + \frac{2}{3}\mu\right)\nabla^2 (\tau_{im} - \alpha P - 3\beta K(T - T_0)) + \nabla F = 0
\]
(A9)

The coefficient of the third term in Eq. (A9) is only a function of Poisson’s ratio \(\nu\)
\[
\frac{\lambda + 2\mu}{\lambda + (2/3)\mu} = \frac{3(1-\nu)}{(1+\nu)}
\]
(A10)

Eq. (A9) then becomes
\[
\frac{3(1-\nu)}{(1+\nu)}\nabla^2 \tau_m + \nabla F \left[\frac{2(1-2\nu)}{(1+\nu)}(\alpha\nabla^2 P + 3\beta K\nabla^2 T)\right] = 0
\]
(A11)

Appendix B. Empirical corrections for porosity

B.1. (Zimmerman, 1986) Poroelasticity

A theory of hydrostatic poroelasticity (Zimmerman et al., 1986) has been proposed that accounts for the coupling of rock deformation with fluid flow inside the porous rock. Porous rock has a bulk and a pore volume, and those volumes are acted on by pore pressure and mean stress. The compressibilities are written in terms of those quantities.

\[
C_{bc} = \frac{1}{V_b} \frac{\partial V_b}{\partial \rho_m} \rho_p
\]
(B1)

where subscript \(b\) refers to bulk volume.

Relationships between these compressibilities are derived for an idealized porous medium and from that, dependence of porosity on effective stress

\[
d\phi = -C_{bc}(1-\phi-C_\alpha) d\sigma'
\]
(B2)

where \(C_r\) is rock grain compressibility, an expression for the Biot’s coefficient.

\[
C_r = \frac{1}{C_{bc}}
\]
(B3)

and dependence of bulk volume on pore pressure and effective stress

\[
dV_b = -V_b C_{bc} d\sigma' + C_r dP
\]
(B4)

B.2. Rutqvist et al. (2002), sedimentary rock

Rutqvist et al. (2002) presented the following function for porosity, obtained from laboratory experiments on sedimentary rock by Davies and Davies (1999)

\[
\phi = \phi_r + (\phi_0 - \phi_r)e^{-\sigma'/\sigma_d}
\]
(B5)

where \(\phi_0\) is zero effective stress porosity, \(\phi_r\) is high effective stress porosity, and the exponent \(\alpha\) is a parameter.

B.3. Rutqvist et al. (2002), fractures

For fractured media, they defined an aperture width \(b_k\) for direction \(k\) as

\[
b_k = b_{0,k} + \Delta b_k(e^{-\sigma'/\sigma_d} - e^{-\sigma'/\sigma_0}), k = x, y, z
\]
(B6)

where subscript 0 refers to initial conditions, \(\Delta b_k\) is the aperture change, and the exponent \(d\) is a parameter. Porosity is correlated to changes in \(b_k\) as

\[
\phi = \phi_0 \frac{b_1 + b_2 + b_3}{b_{1,0} + b_{2,0} + b_{3,0}}
\]
(B7)


McKee et al. (1988) derived a relationship between porosity and effective stress from hydrostatic poroelasticity theory by assuming incompressible rock grains

\[
\phi = \phi_0 \frac{e^{\sigma'/\sigma_d}}{1-\phi_0 (1-e^{\sigma'/\sigma_d})}
\]
(B8)

where \(C_p\) is average pore compressibility.

Appendix C. Permeability correlations

C.1. Rutqvist et al. (2002), sedimentary rock

An associated function for permeability in terms of porosity is

\[
k = k_0 e^{c(\phi - \phi_d) - 1}
\]
(C1)

where \(k_0\) is zero stress permeability and the exponent \(c\) is a parameter.

C.2. Rutqvist et al. (2002), fractures

Direction \(k\) permeability is correlated to fracture aperture of other directions \(l\) and \(m\) as

\[
k_k = \frac{b_{l,k}^2 + b_{m,k}^2}{b_{l,0}^2 + b_{m,0}^2}
\]
(C2)

C.3. Carman–Kozeny

A relationship between permeability and effective stress was obtained from the Carman–Kozeny equation

\[
k_0 \frac{\phi^d}{(1-\phi)^{\nu}}
\]
(C3)

and the above relationship for porosity. These relationships fit laboratory and field data for granite, sandstone, clay, and coal.


Ostensen (1986) studied the relationship between effective stress and permeability for tight gas sands and approximated permeability as

\[
k^n = D \ln \frac{\sigma^*}{\sigma}
\]
(C4)

where \(n = 0.5, D\) is a parameter, and \(\sigma^*\) is effective stress for zero permeability, obtained by extrapolating measured square root permeability versus effective stress on a semi-log plot.


Verma and Pruess (1988) presented a power law expression relating permeability to porosity

\[
\frac{k-k_c}{k_0-k_c} = \left(\frac{\phi - \phi_c}{\phi_0 - \phi_c}\right)^n
\]
(C5)

where \(k_c\) and \(\phi_c\) are asymptotic values of permeability and porosity, respectively, and exponent \(n\) is a parameter.
Appendix D. Analytical solutions for 1D consolidation problem

The analytical solution for the 1-D consolidation problem follows:

\[ P(z, t) = \sum_{n=1, \ldots, \infty} \frac{4}{\pi n} \sin \left( \frac{\pi n z}{2b} \right) \exp \left( - \frac{n^2 \pi^2 kt}{4\mu s^2} \right) \]  \hspace{1cm} (D1)

Vertical displacement of the upper surface is

\[ w(z=0, t) = \frac{\alpha_p h}{\alpha_p + 2G} \left[ 1 - \frac{\alpha_p^2 M}{\alpha_p + 2G + \alpha_p^2 M} \sum_{n=1, \ldots, \infty} \frac{8}{\pi n^2} \exp \left( - \frac{n^2 \pi^2 kt}{4\mu s^2} \right) \right] \]  \hspace{1cm} (D2)

where

\[ P_0 = \frac{\alpha_p M}{\alpha_p + 2G + \alpha_p^2 M} \sigma_{ex} \]  \hspace{1cm} (D3)

\[ M = \frac{1}{\phi c_t} \]  \hspace{1cm} (D4)

\[ S = \frac{1}{M} + \frac{\alpha_p^2}{\alpha_p + 2G} \]  \hspace{1cm} (D5)

Appendix E. Analytical solutions for 1D heat conduction problem

The analytical solution for the 1-D heat conduction problem follows:

Temperature during the cooling is:

\[ T(z, t) = T_0 + (T_1 - T_0)erfc \left( \frac{z}{\sqrt{4D_1 t}} \right) \]  \hspace{1cm} (E1)

The vertical displacement is

\[ w(z=0, t) = -\frac{(1+\phi)(T_1 - T_0)}{(1+\phi)} \left[ \frac{h}{\sqrt{4D_1 t}} + \frac{\exp \left( -h^2/(4D_1 t) \right) - 1}{\sqrt{\pi}(4D_1 t)} \right] \]  \hspace{1cm} (E2)

Appendix F. Analytical solutions for Mandel’s problem

The original Mandel’s solutions (1953) provides only the analytical form for the pore pressure. Later, Abousleiman et al. (1996) extend the solution to all field quantities for materials with transverse isotropy, as well as compressible pore fluid and solid constituents. The solutions are given as the following.

Pressure solution

\[ p(x,t) = \frac{2F}{\sum_{i=1}^{d}} \frac{\sin(\psi)}{\sinh(\psi)} \left[ \psi \left( \frac{\sin(\psi x/a)}{\sinh(\psi)} \right) \exp \left( - \frac{\psi^2 t}{\alpha^2} \right) \right] \]  \hspace{1cm} (F1)

where \( d \) is dimension of specimen, \( 2F \) is force applied to the top of the specimen (Pa), \( \psi \) is an infinite series defined by \( \tan \psi / \psi = A_1 / A_2 \), \( x \) is location of interest (m), \( t \) is time (s)

\[ A_1 = \frac{\alpha_p^2 M_{13} - 2\alpha_1 \alpha_p M_{11} + \alpha_p^2 M_{11}}{\alpha_3 M_{13} - \alpha_p M_{11}} + \frac{M_{11} M_{13} - M_{11}^2}{M_{11} (M_{13} - \alpha_1 M_{11})} \]  \hspace{1cm} (F2)

\[ A_2 = \frac{\alpha_p M_{11} - \alpha_1 M_{13}}{M_{11}} \]

where \( \alpha_1 \) is Biot constant of direction QUOTE and \( M_{11} \) is drained elastic modulus defined as

\[ M_{11} = M_{13} = \frac{E(1-\nu)}{1-2\nu} \]

and, \( c_1 \) is fluid flow and mechanical properties of the specimen defined as

\[ c_1 = \frac{k_1 M_{11}}{\mu M_{11}^2} \]

where \( k_1 \) is permeability in the x-direction, \( M \) is the Biot modulus defined as \( (\phi c_t)^{-1} \), \( \mu \) is fluid viscosity, and \( M_{11} \) is undrained elastic modulus in the x-direction defined as \( M_{11} = M_{11} + \alpha_1^2 M_{11} \)

Displacement solutions:

x-direction:

\[ u_x(x,t) = -\frac{F}{\alpha M_{13} M_{33} - \alpha_1 M_{13} + M_{11}} \sum_{i=1}^{d} \left[ \frac{\sin(\psi_x) \cos(\psi_x)}{\sinh(\psi)} \exp \left( - \frac{\psi^2 t}{\alpha^2} \right) \right] \]  \hspace{1cm} (x)

\[ -\frac{2F \alpha_1}{\alpha_1 M_{13} M_{33} - \alpha_1 M_{13} + M_{11}} \sum_{i=1}^{d} \left[ \frac{\cos(\psi_x) \sin(\psi_x) \cos(\psi_x)}{\sinh(\psi)} \exp \left( - \frac{\psi^2 t}{\alpha^2} \right) \right] \]  \hspace{1cm} (F)

z-direction:

\[ u_z(z,t) = \frac{F}{\alpha M_{13} M_{33} - \alpha_1 M_{13} + M_{11}} \left( \frac{1 + 2 \alpha_2}{\alpha_2} \right) \sum_{i=1}^{d} \left[ \frac{\sin(\psi_z) \cos(\psi_z)}{\sinh(\psi)} \exp \left( - \frac{\psi^2 t}{\alpha^2} \right) \right] \]  \hspace{1cm} (E)

Volumetric strain

\[ n = 1 - \left( 1 - u_x/x \right) \left( 1 - u_y/y \right) \]

References


